Dirac cones for cold atoms in an optical lattice

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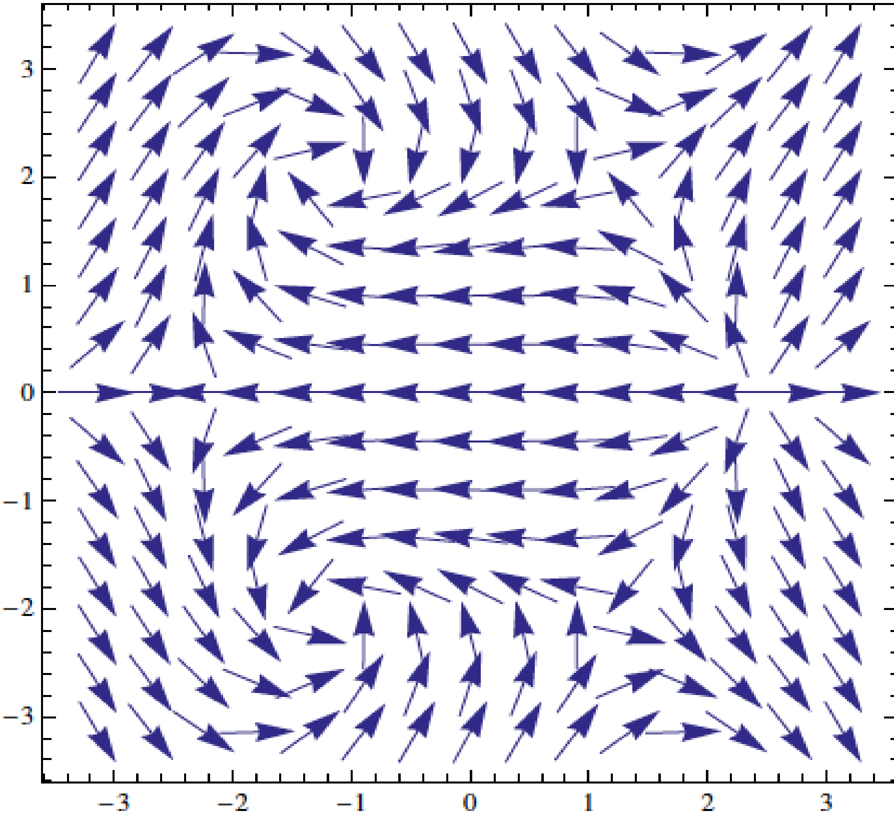

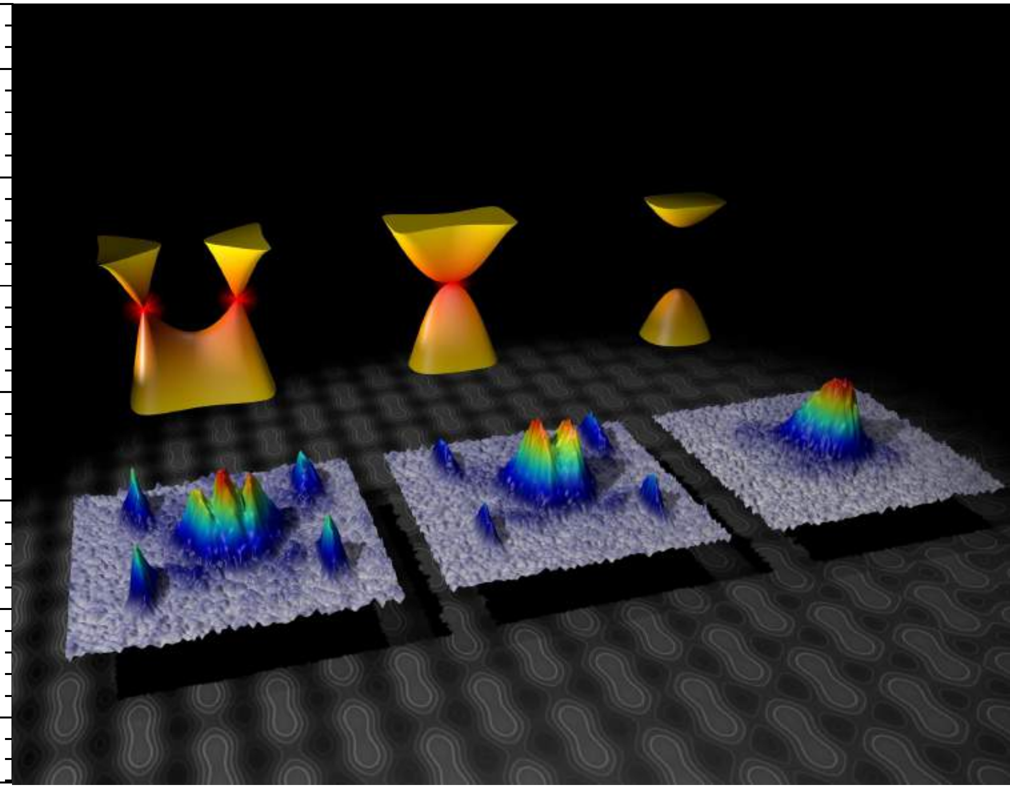

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"Hadrons and nuclear physics meet cold atoms", IHP Paris, 02/02/2018

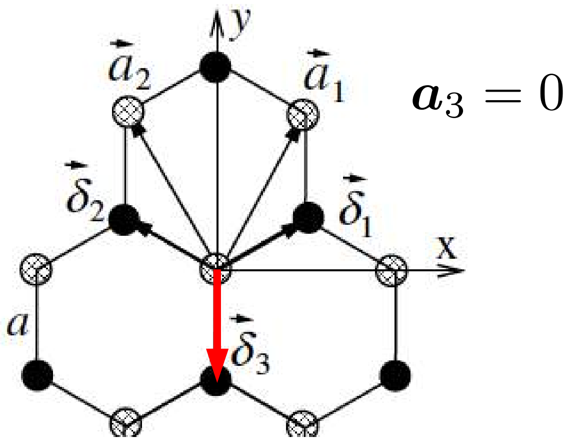
Outline

1) Lattice simulation of relativistic particles :
quantum particle in a periodic lattice
→ emergent Dirac or Weyl equation at low energy
e.g. electrons in a crystal (graphene)
Cold atoms in an optical lattice (honeycomb or brick-wall)

2) Band contact points (e.g. Dirac/Weyl point) as topological defects

3) Probing these topological defects via Stückelberg interferometry for cold atoms in optical lattices

Tight-binding model on honeycomb



Simplest tight-binding model of graphene
Wallace PR 1947

$$H(\mathbf{k})|u_{\mathbf{k}}\rangle = \varepsilon_{\mathbf{k}}|u_{\mathbf{k}}\rangle$$

$$|u_{\mathbf{k}}\rangle = \begin{pmatrix} u_{A\mathbf{k}} \\ u_{B\mathbf{k}} \end{pmatrix}$$

Sub-lattice spinor (amplitude on A and B sites)

$$H(\boldsymbol{k}) = \begin{pmatrix} 0 & f(\boldsymbol{k})^* \\ f(\boldsymbol{k}) & 0 \end{pmatrix} = |f|(\cos\phi\sigma_x + \sin\phi\sigma_y)$$

Phase = azimuthal angle along equator of Bloch sphere

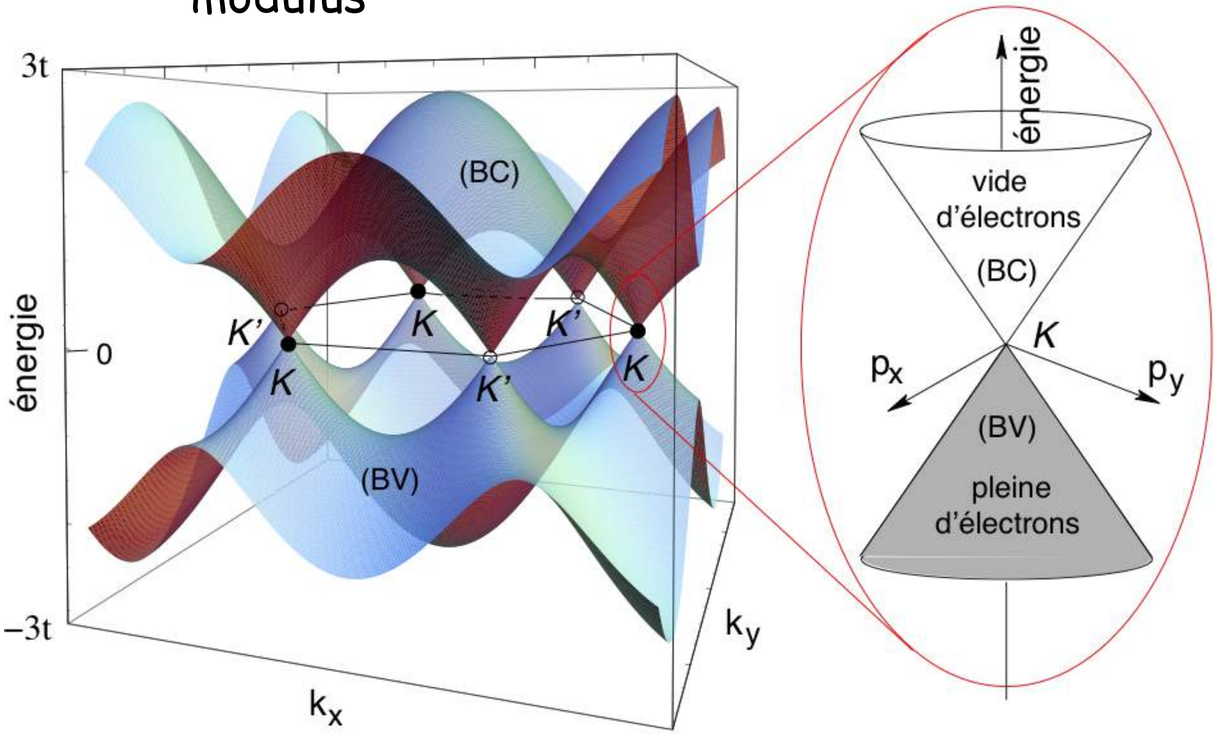
$$f(\boldsymbol{k}) = -t\sum_{j=1}^{3} e^{-i\boldsymbol{k}\cdot\boldsymbol{\delta}_j} = |f(\boldsymbol{k})|e^{i\phi(\boldsymbol{k})}$$

Modulus and phase

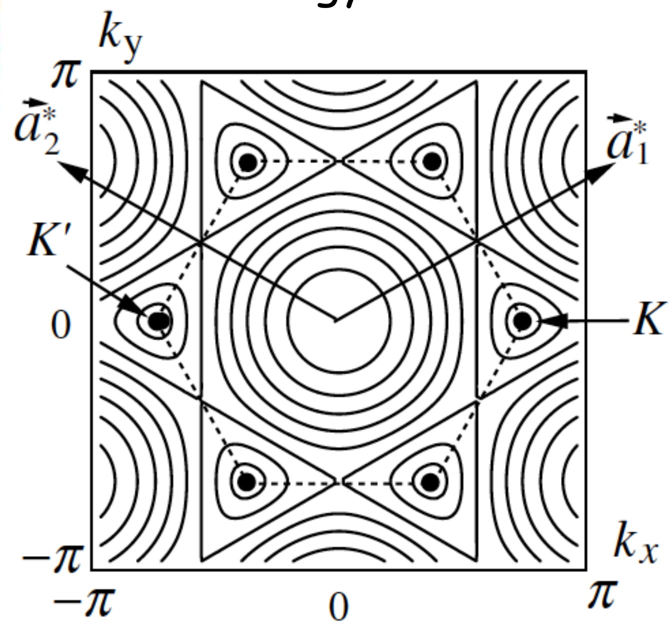
Modulus → energy spectrum

$$\varepsilon_{\mathbf{k}} = \pm|f(\mathbf{k})| = \pm t\sqrt{3 + 2\cos(\mathbf{k}\cdot\mathbf{a}_1)2\cos(\mathbf{k}\cdot\mathbf{a}_2) + 2\cos(\mathbf{k}\cdot(\mathbf{a}_1 - \mathbf{a}_2))}$$

modulus



Hexagonal Brillouin zone
Iso-energy contours



Phase → eigenstate & quantized vortex

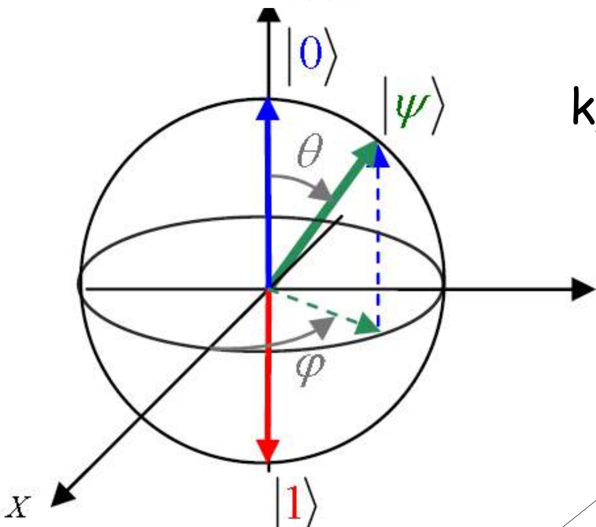
$$|u_{\mathbf{k}}\rangle = \begin{pmatrix} u_{A\mathbf{k}} \\ u_{B\mathbf{k}} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \pm e^{i\phi_{\boldsymbol{k}}} \end{pmatrix}$$

Cell-periodic Bloch state (upper/lower band)

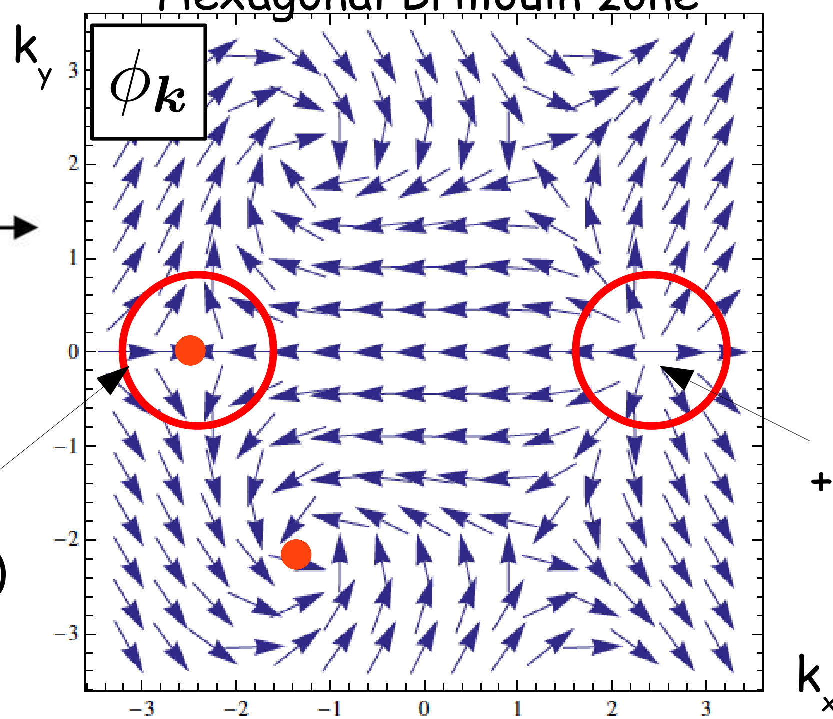
Phase = azimuthal angle along equator of Bloch sphere

$$\Gamma(C) = \oint_C d\mathbf{k} \cdot i\langle u_{\mathbf{k}} | \nabla_{\mathbf{k}} u_{\mathbf{k}} \rangle = \frac{1}{2} \oint_C d\phi_{\boldsymbol{k}} = \pm\pi$$

Berry phase (here quantized, winding number x π)



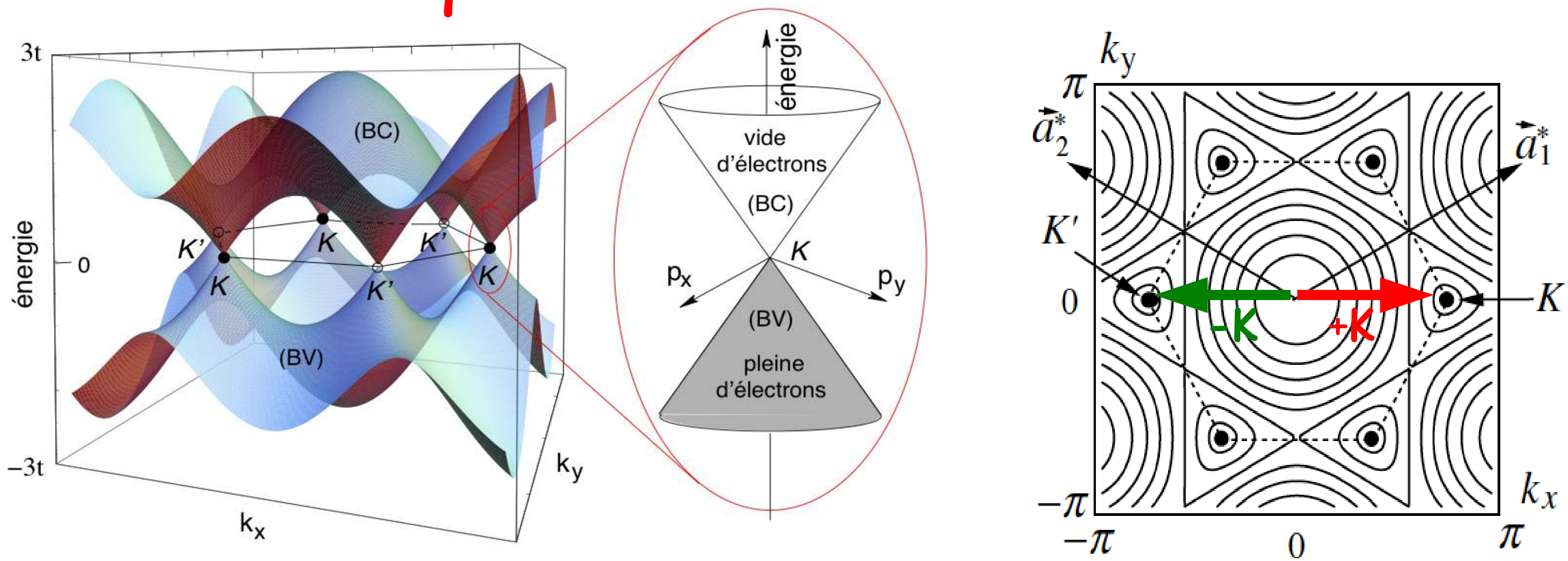
Hexagonal Brillouin zone



-1 (anti-vortex)
K' point

+1 (vortex)
K point

Dirac equation: 2+1 and massless



$\mathbf{k} = \xi \mathbf{K} + \mathbf{q}$ where $\xi = \pm 1$ is the valley index K/K'

$$H(\mathbf{k}) \approx H_\xi(\mathbf{q}) = \hbar v_F (\xi q_x \sigma_x + q_y \sigma_y) = \hbar v_F \begin{pmatrix} 0 & \xi q_x - i q_y \\ \xi q_x + i q_y & 0 \end{pmatrix}$$

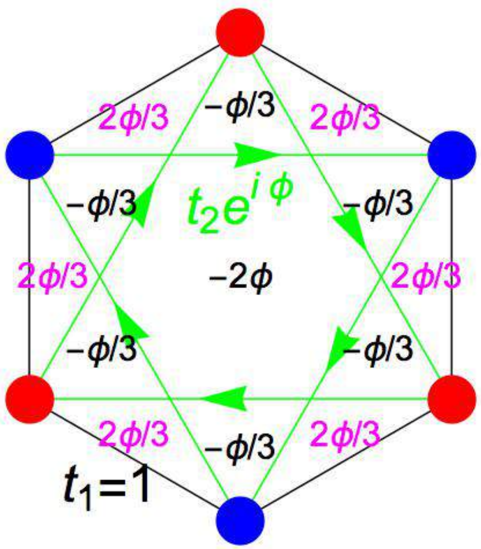
$v_F \equiv \frac{3}{2}\frac{ta}{\hbar} \approx 10^6 \text{ m/s}$ is the Fermi velocity (density independent)

Pauli matrices in sublattice A/B sub-space
2D massless Dirac (Weyl) Hamiltonian in two copies (valley degeneracy)
Four copies if (real) spin is included

Semenoff; di Vincenzo & Mele 1984

Haldane's model of Chern insulator

Spinless graphene with inhomog. B-field (broken TRS) within unit cell of honeycomb lattice + NNN hopping $t_2 e^{i\phi}$



$$H(\boldsymbol{k}) \approx H_G(\boldsymbol{k}) + \Delta_H(\boldsymbol{k})\sigma_z$$

$$\Delta_H(\boldsymbol{k}) = 2t_2 \sin\phi \sum_{j=1}^{3} \sin(\boldsymbol{k}\cdot\boldsymbol{b}_j)$$

$$\Delta_H(K/K') = \pm t_2 3\sqrt{3}\sin\phi$$

Pair of massive Dirac fermions with opposite masses

$$H_H(\boldsymbol{q}) = (\tau_z q_x \sigma_x + q_y \sigma_y)1_s + \Delta_H \tau_z \sigma_z \rightarrow \varepsilon = \pm\sqrt{\boldsymbol{q}^2 + \Delta_H^2}$$

Gap/mass changes sign with valley

- Bulk energy spectrum similar to boron nitride

$$\text{Gap} \;= 2|\Delta_H| = 2t_2 3\sqrt{3}|\sin\phi|$$

- Berry curvature ≥ 0 → Chern ≠ 0 → QHE without LLs
- Edge energy spectrum ?

Haldane 1988

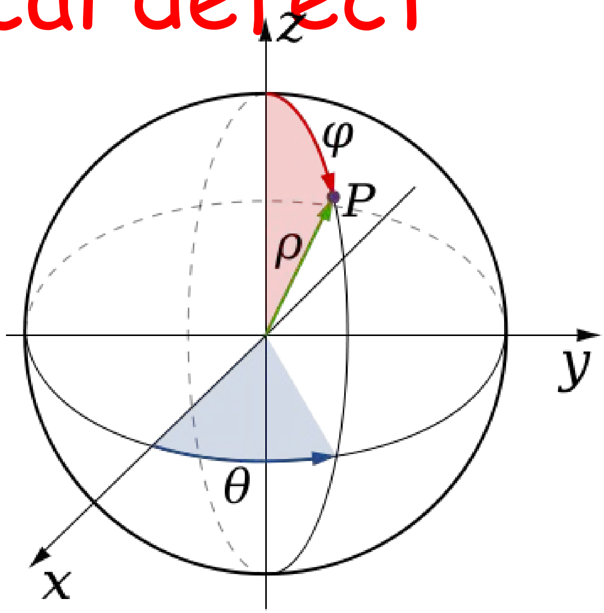
Band contact as topological defect

Band contact between two bands:

$$H(\mathbf{k}) = \epsilon_0(\mathbf{k})\sigma_0 + \vec{h}(\mathbf{k}) \cdot \vec{\sigma}$$

$$\epsilon_\pm(\mathbf{k}) = \epsilon_0(\mathbf{k}) \pm |\vec{h}(\mathbf{k})|$$

$$\vec{h}(\mathbf{k}) = |\vec{h}(\mathbf{k})|\vec{n}(\mathbf{k})$$ Bloch sphere S^2



Band contact : $|\vec{h}(\mathbf{k})| = 0$ → 3 constraints (3 Pauli matrices)

$$h_j(\mathbf{k}) = 0, \;\; j = 1, 2, 3 = x, y, z$$

How many independent parameters?

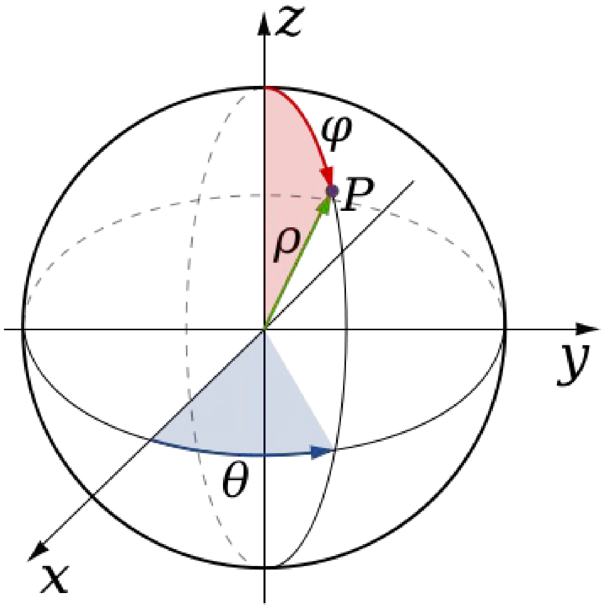
Von Neumann – Wigner 1929

Band contact as topological defect

D=3 → 3 k parameters → stable contact : Weyl point

D=2 → 2 k parameters → unstable contact

D=2 with symmetry constraint (such as T and I or chiral S)
→ only 2 Pauli matrices (motion on equator S^1)
→ stable contact: Dirac points in graphene



Homotopy groups:

Encircling of the defect: S^{D-1}

Target space: Bloch sphere S^2 or equator S^1

$S^2 \to S^2$	$\Pi_2(S^2) = \mathbb{Z}$	Wrapping number
$S^1 \to S^2$	$\Pi_1(S^2) = 0$	Cannot lasso a basketball
$S^1 \to S^1$	$\Pi_1(S^1) = \mathbb{Z}$	Winding number

Band contact as topological defect

2D Dirac points : linear band contact points of winding W=±1
(Berry phase = π)

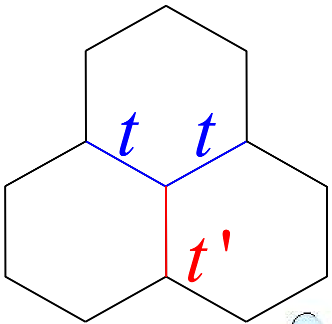
Dirac points appear in pairs of opposite winding (chirality)
Fermion doubling (Nielsen-Ninomiya theorem)

Motion on an equator (great circle) of Bloch sphere
$\rightarrow$ single angle $\phi(\mathbf{k})$
Quantized vortices (vorticity = winding number)

Other band contacts exist in 2D :
Semi-Dirac (linear-quadratic) W=0 (Berry phase = 0)
Quadratic band contact W=0 (Berry phase = 0)
Quadratic band contact W=±2 (Berry phase = 0)

Conservation of total winding number during annihilation/creation of topological defects

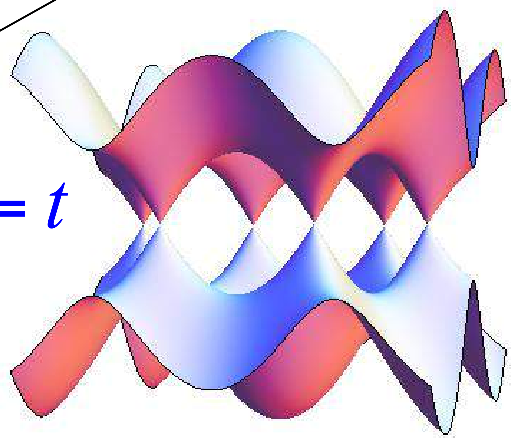
Merging transition



$$\varepsilon = \pm|t' + te^{i\vec{k}\cdot\vec{a}_1} + te^{i\vec{k}\cdot\vec{a}_2}|$$

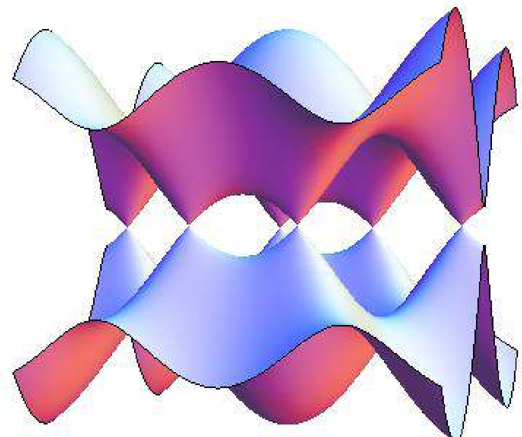
t-t' model

$t' = t$



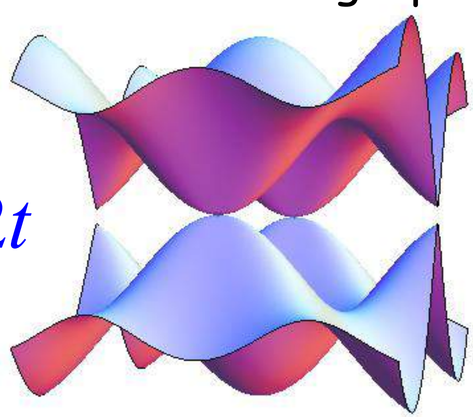
Undeformed graphene

$t' = 1.5t$



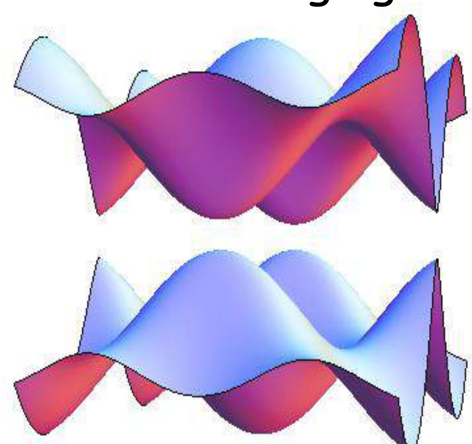
Before merging: Dirac cones

$t' = 2t$



Merging point: semi-Dirac

$t' = 2.3t$

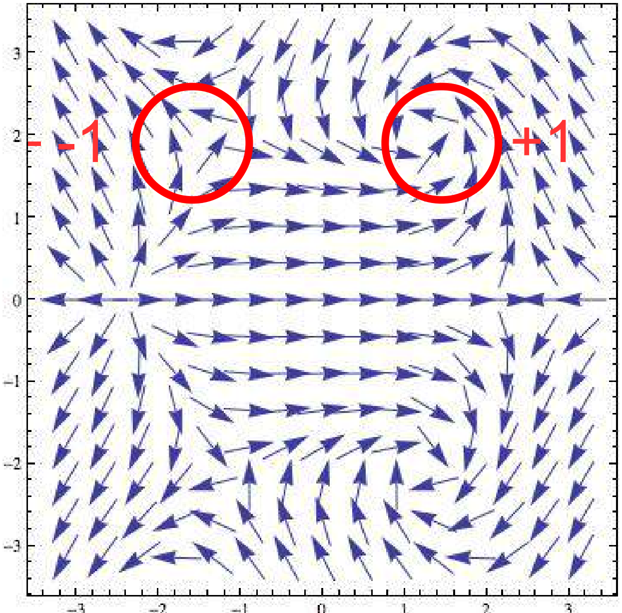


After merging: gapped

Lifshitz transition = change in topology of the Fermi surface

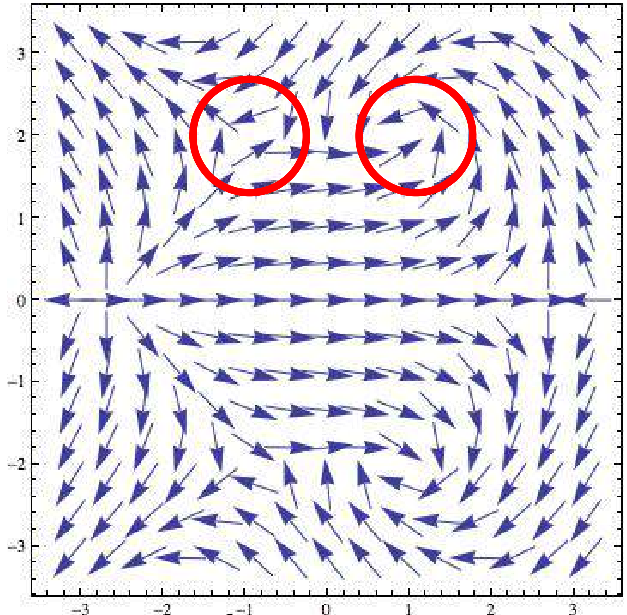
Annihilation of vortices

$t' = t$



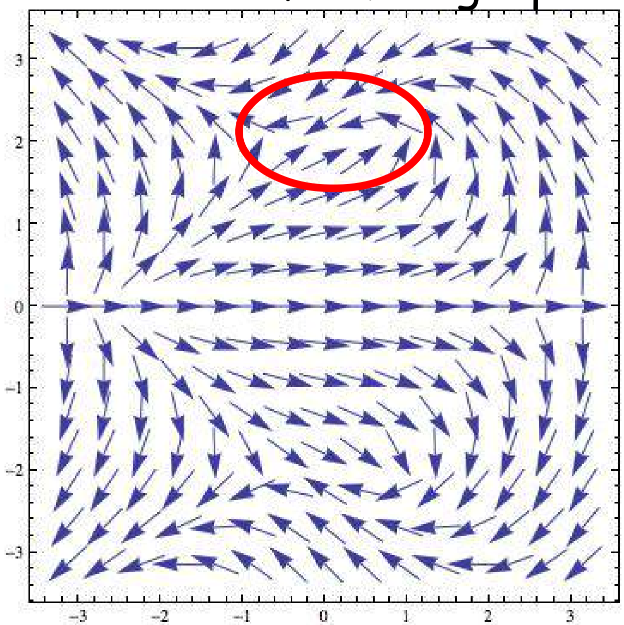
Undeformed graphene

$t' = 1.5t$



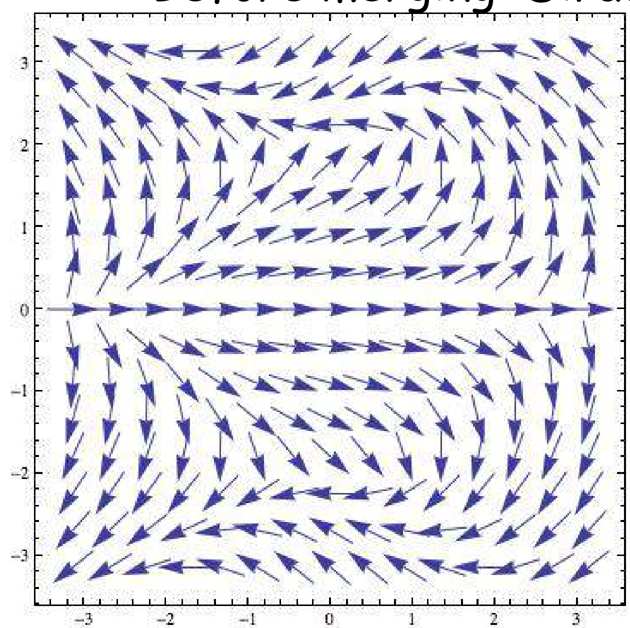
Before merging: Dirac cones

$t' = 2t$



Merging point: semi-Dirac

$t' = 2.3t$

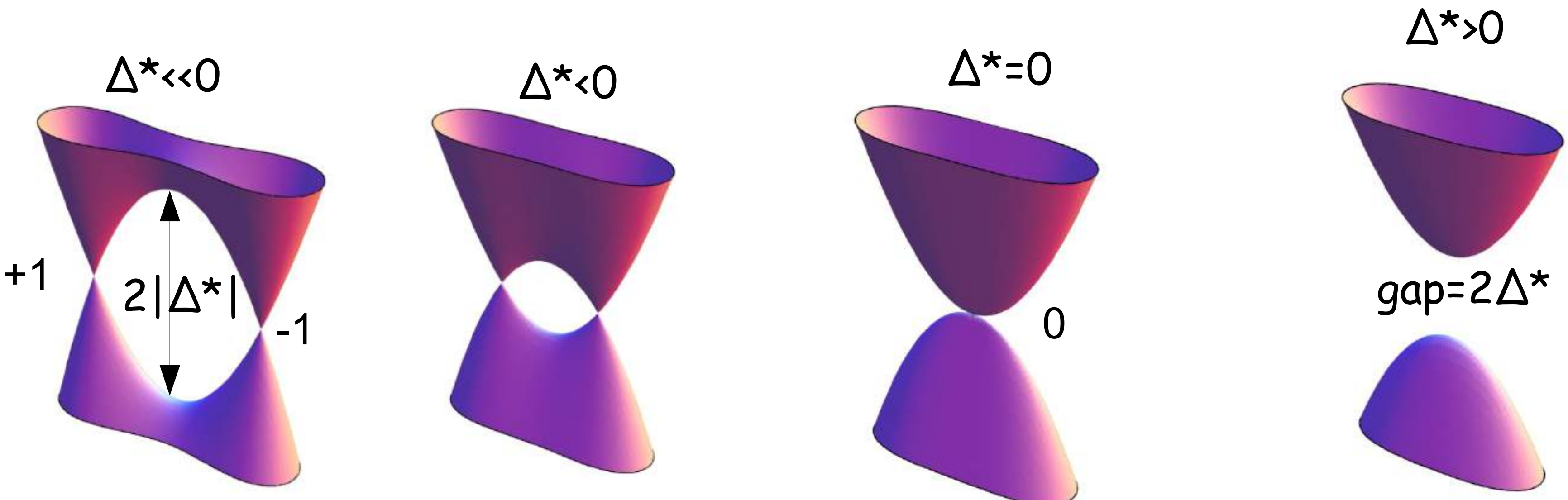


After merging: gapped

« Universal » Hamiltonian of +- merging

$$\mathcal{H}(\boldsymbol{q}) = \begin{pmatrix} 0 & \Delta^* + \frac{q_x^2}{2m^*} - i c_y q_y \\ \Delta^* + \frac{q_x^2}{2m^*} + i c_y q_y & 0 \end{pmatrix}$$

$$\varepsilon = \pm\sqrt{\left(\Delta^* + \frac{q_x^2}{2m^*}\right)^2 + (c_y q_y)^2}$$

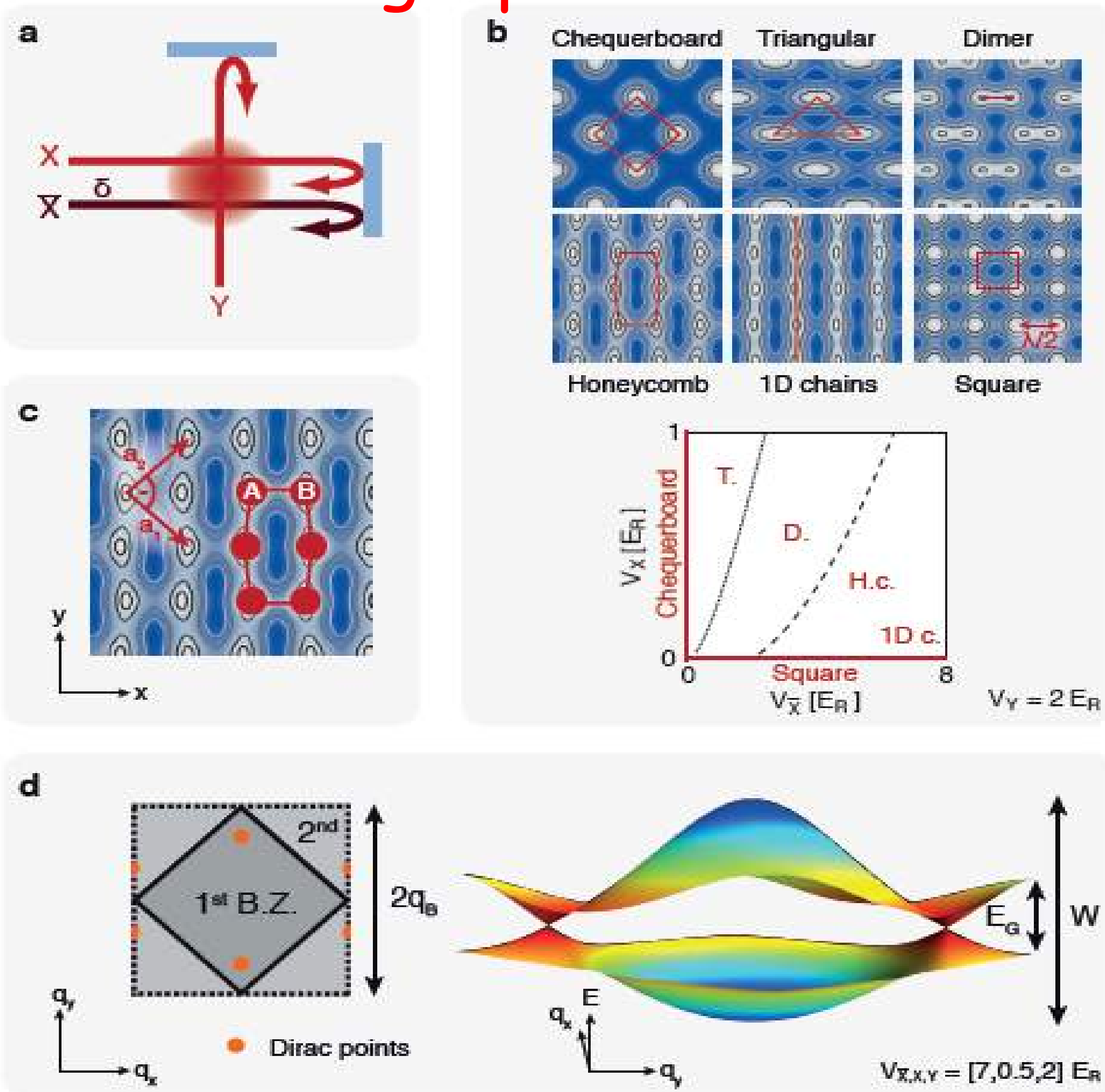


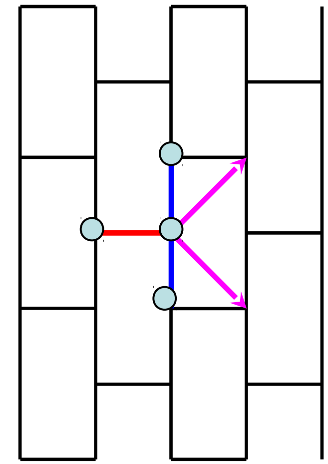
Pair of Dirac points
(+-)

semi-Dirac

Montambaux, Piéchon, Fuchs, Goerbig PRB 2009

Artificial graphene with cold atoms



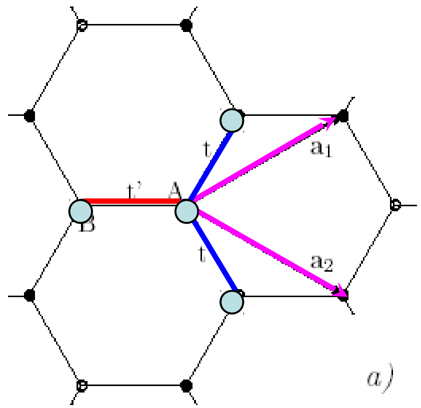


Tarruell, Greif, Uehlinger, Jotzu & Esslinger Nature 2012

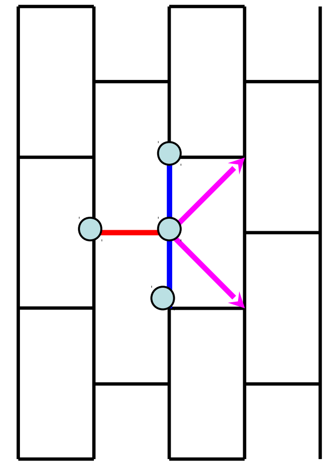
Artificial graphene VS Real graphene

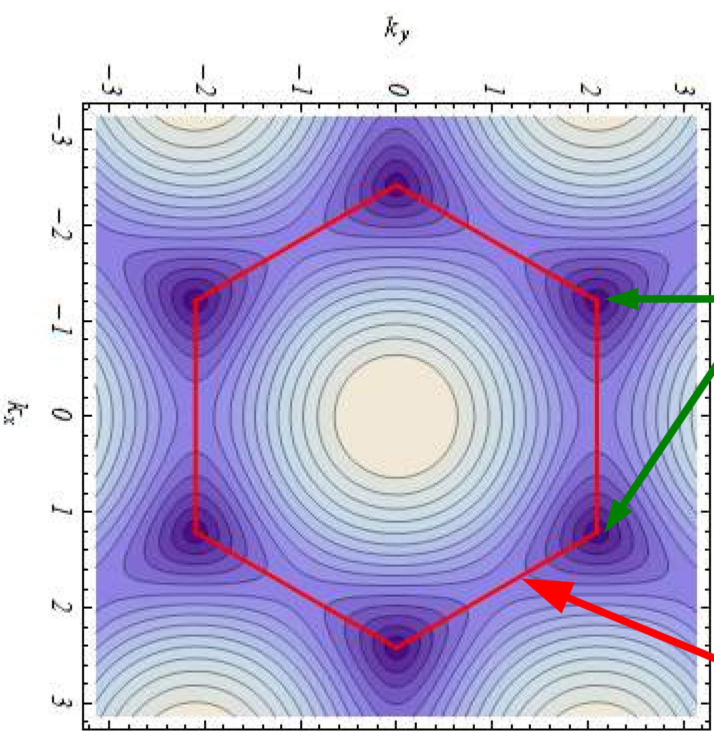
Artificial graphene	Real graphene
Fermionic atoms: spin polarized, neutral, no s-wave interaction	Conduction electrons: spin $\frac{1}{2}$, Coulomb repulsion
Tunable optical lattice (honeycomb-like)	Carbon honeycomb lattice
Band filling: number of atoms Degenerate ideal Fermi gas ($T \sim 0.1\ \varepsilon_F$).	Band filling: backgate controls chemical potential ε_F 2D degenerate Fermi liquid.
Inhomogeneous atomic density (trapping potential)	Homogeneous electronic density
No transport measurement, no ARPES: how to « see » the band structure?	Band structure probed directly by ARPES or indirectly in transport

Honeycomb VS brick wall : t-t' model

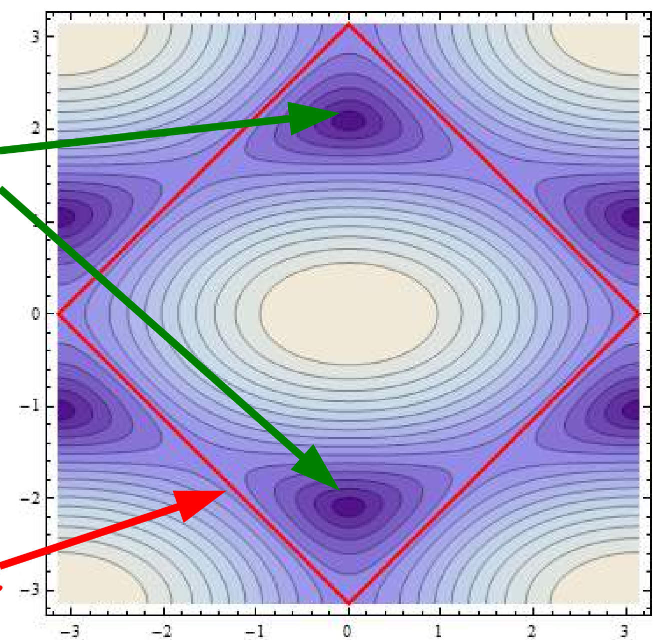


Real space:
t hopping
t' hopping
Bravais lattice vectors





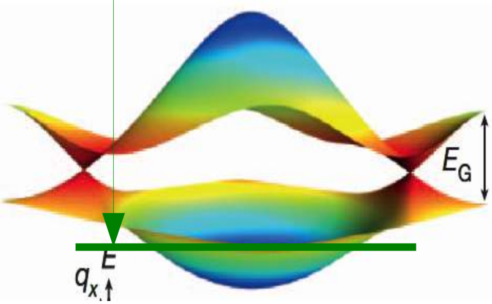
Reciprocal space:
Iso-energy lines
2 Dirac points



First Brillouin zone

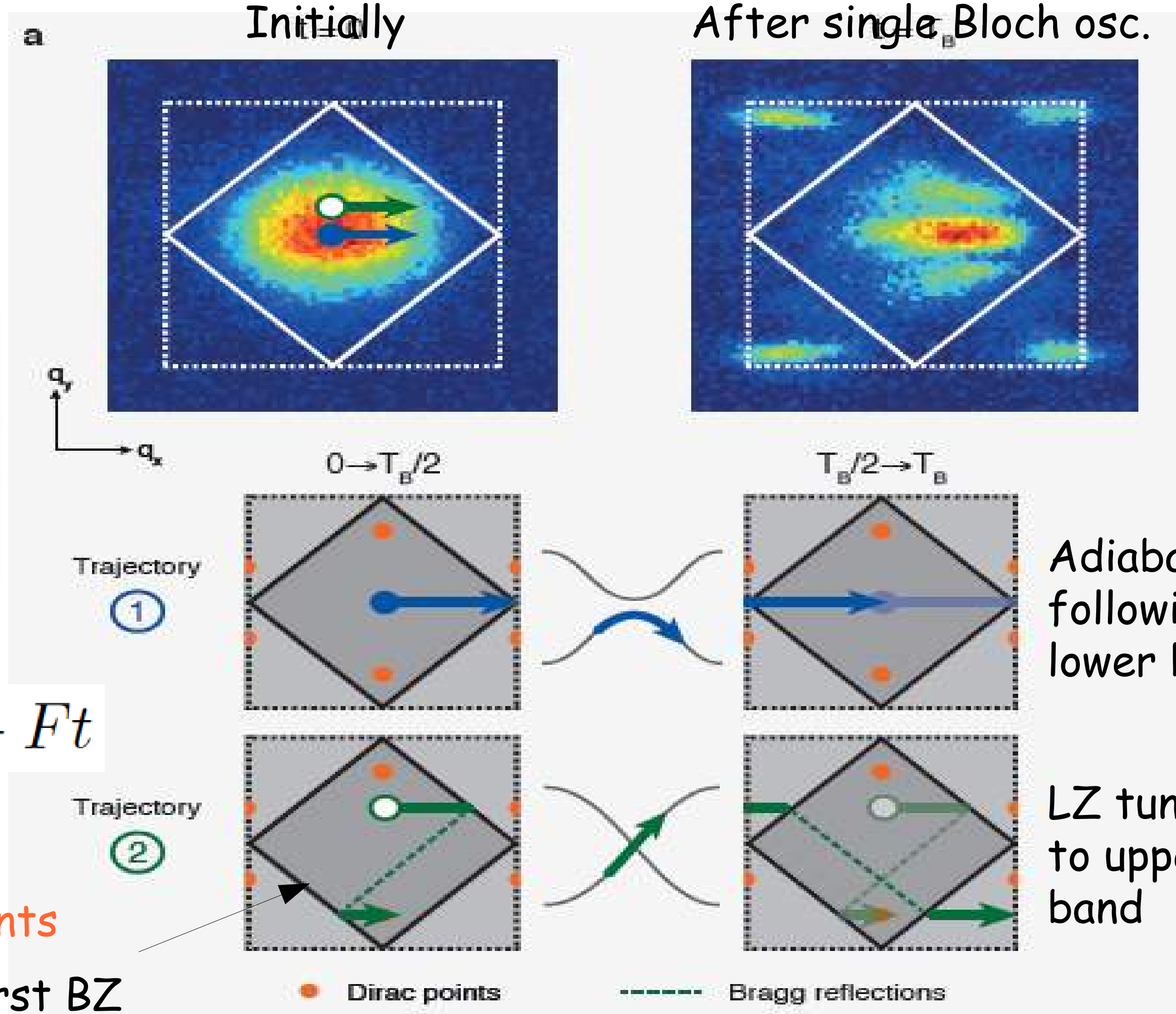
Bloch oscillation, Landau-Zener tunneling and band mapping technique

Initial equilibrium Fermi sea (Fermi level in lower band)



$$k_x(t) = k_x(0) + Ft$$

Dirac points



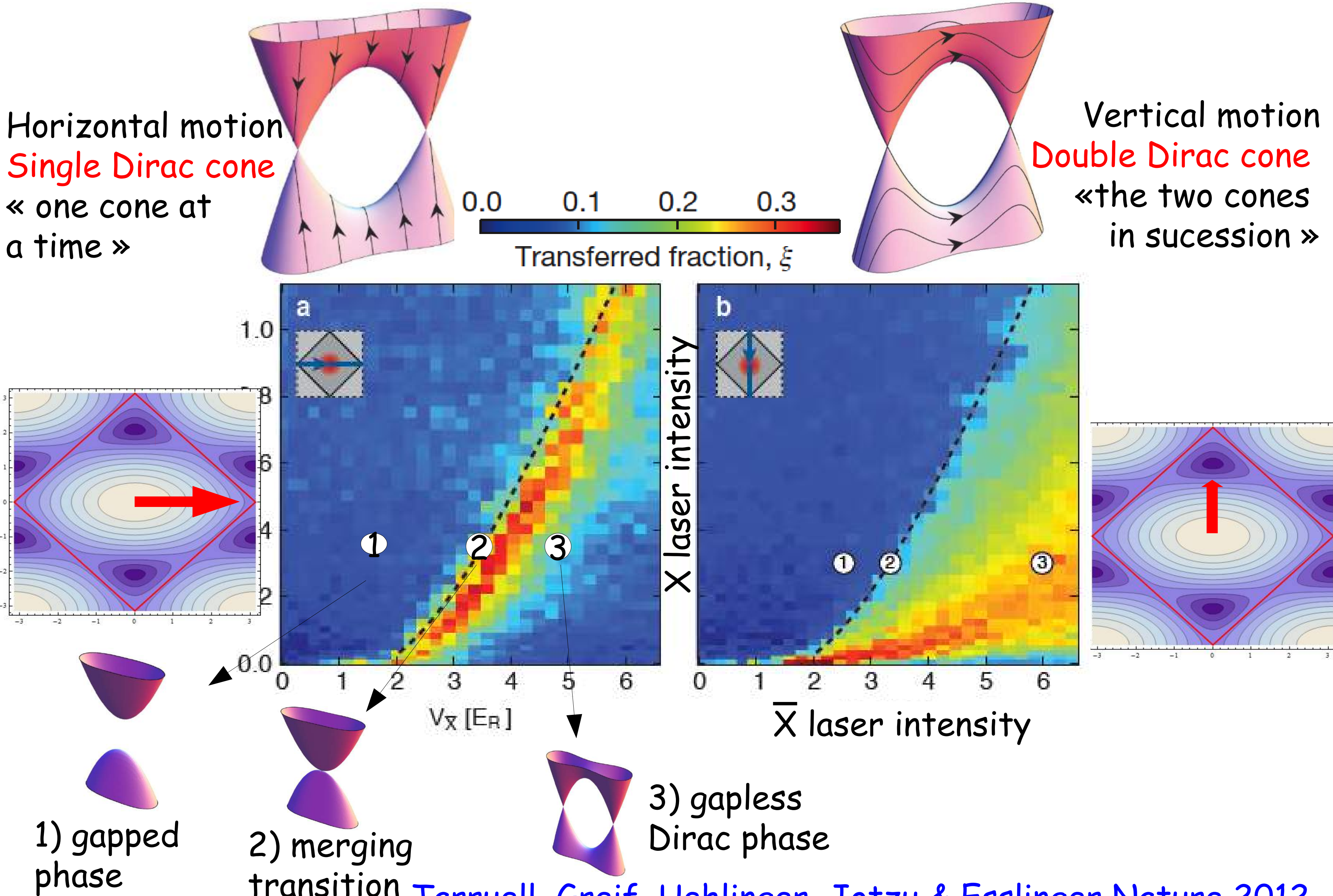
First BZ

Adiabatic following of lower band

LZ tunneling to upper band

Tarruell, Greif, Uehlinger, Jotzu & Esslinger Nature 2012

Transferred fraction after Bloch oscillation

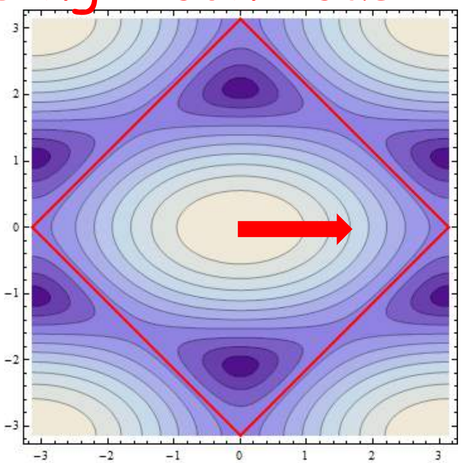


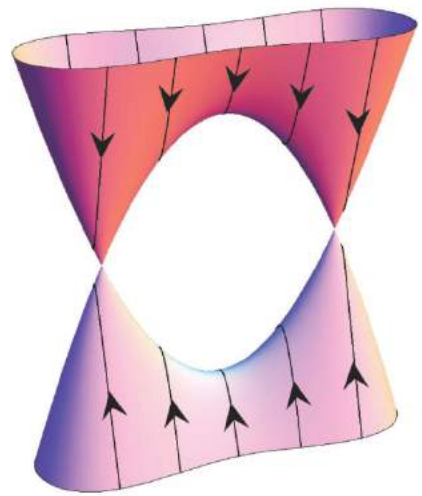
Tarruell, Greif, Uehlinger, Jotzu & Esslinger Nature 2012

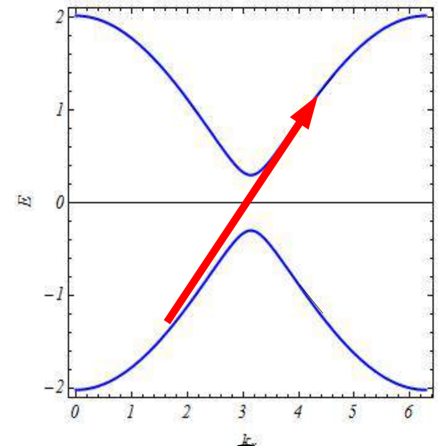
Band changing probability

Landau-Zener probability : $P_Z = e^{-2\pi\delta}$ $\qquad \delta = \dfrac{\text{gap}^2}{\hbar \cdot \text{velocity} \cdot \text{force}}$

Single cone case

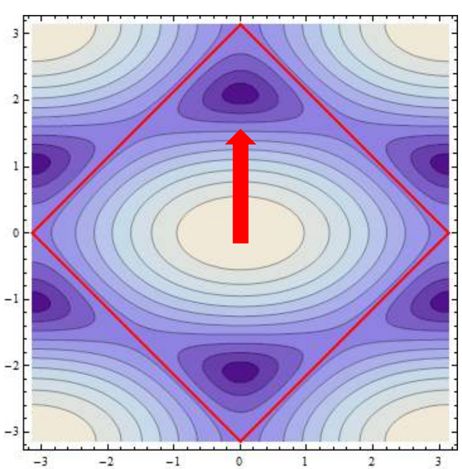


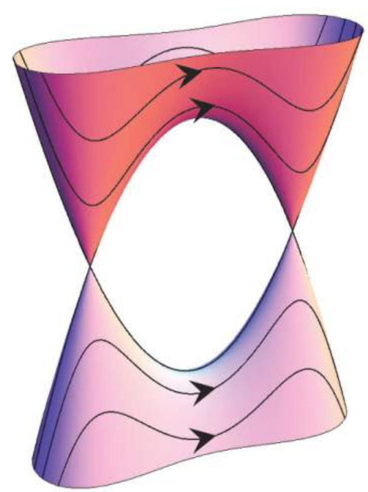


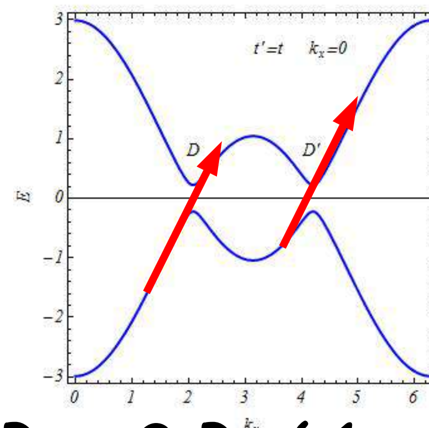


P_Z

Double cone case





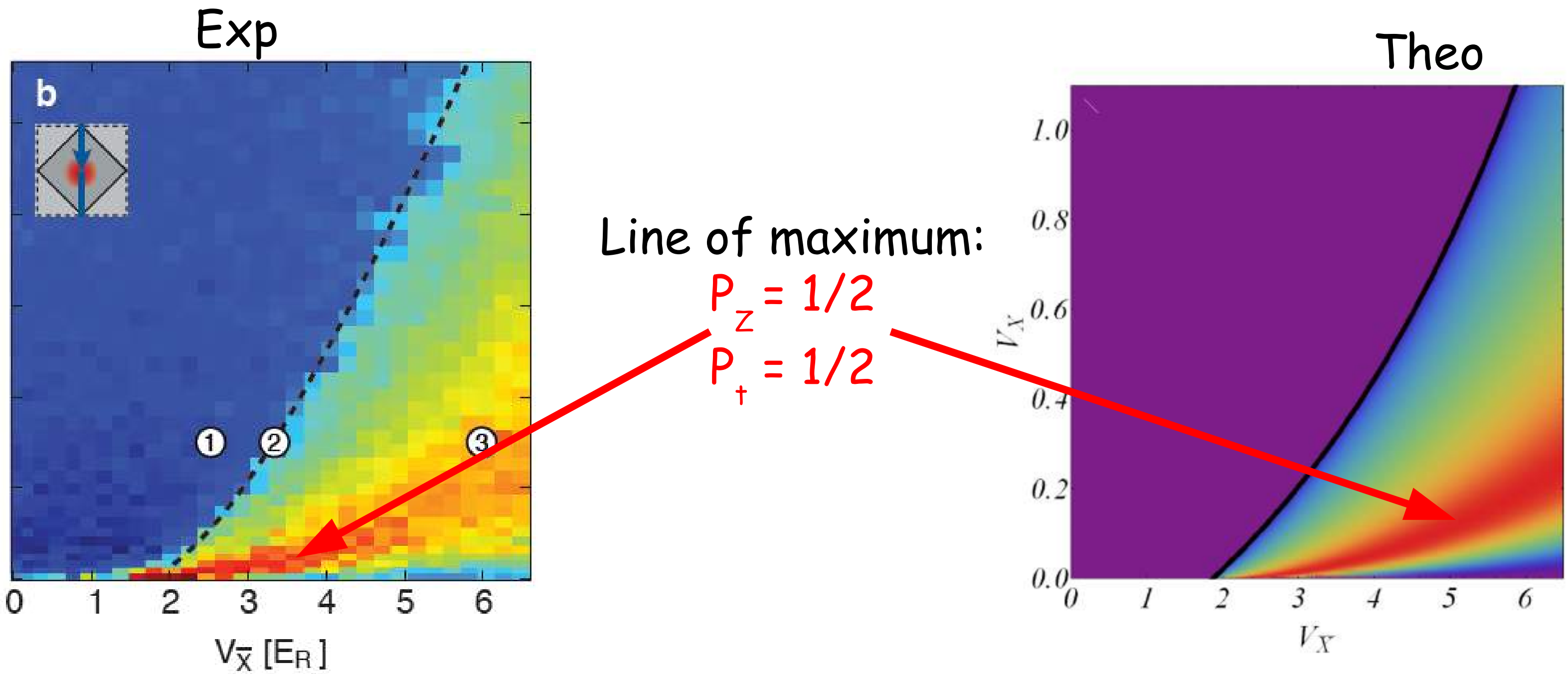


$P_t = 2\, P_Z\, (\,1 - P_Z\,)$

Lim, Fuchs & Montambaux PRL 2012

Double Dirac cone case : $P_t = \langle 2 P_Z (1 - P_Z) \rangle$

Incoherent « interferometer »



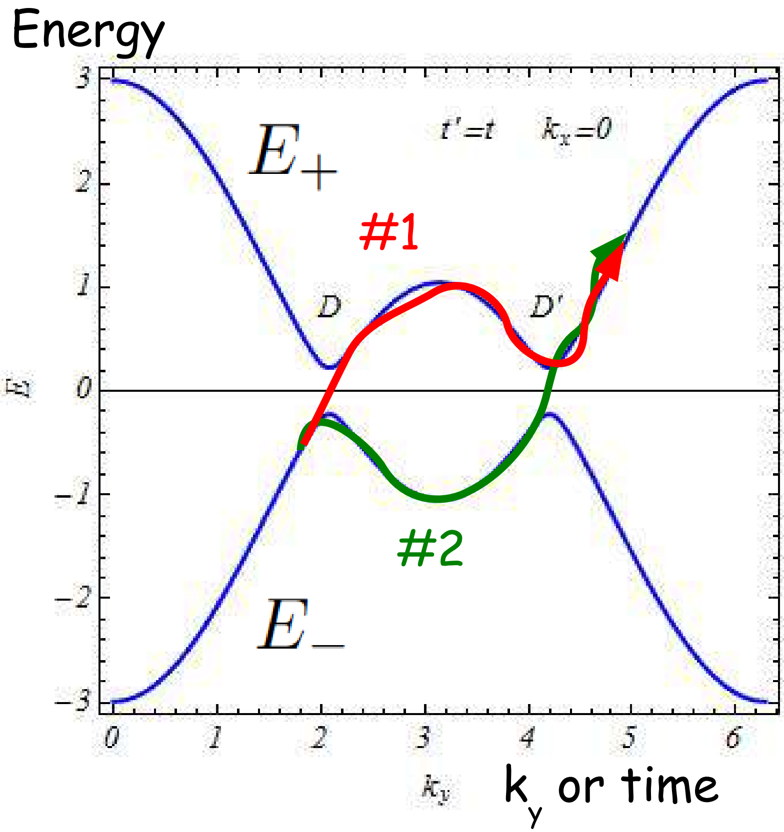
Bloch oscillations + Landau-Zener tunneling + band mapping technique can tell whether Dirac points are present/absent

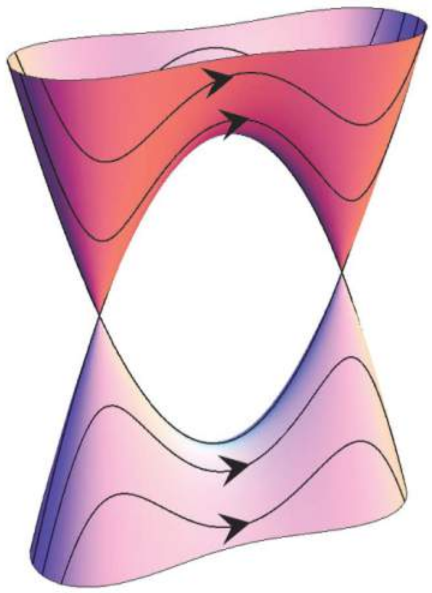
Lim, Fuchs & Montambaux PRL 2012

Stückelberg interferometer

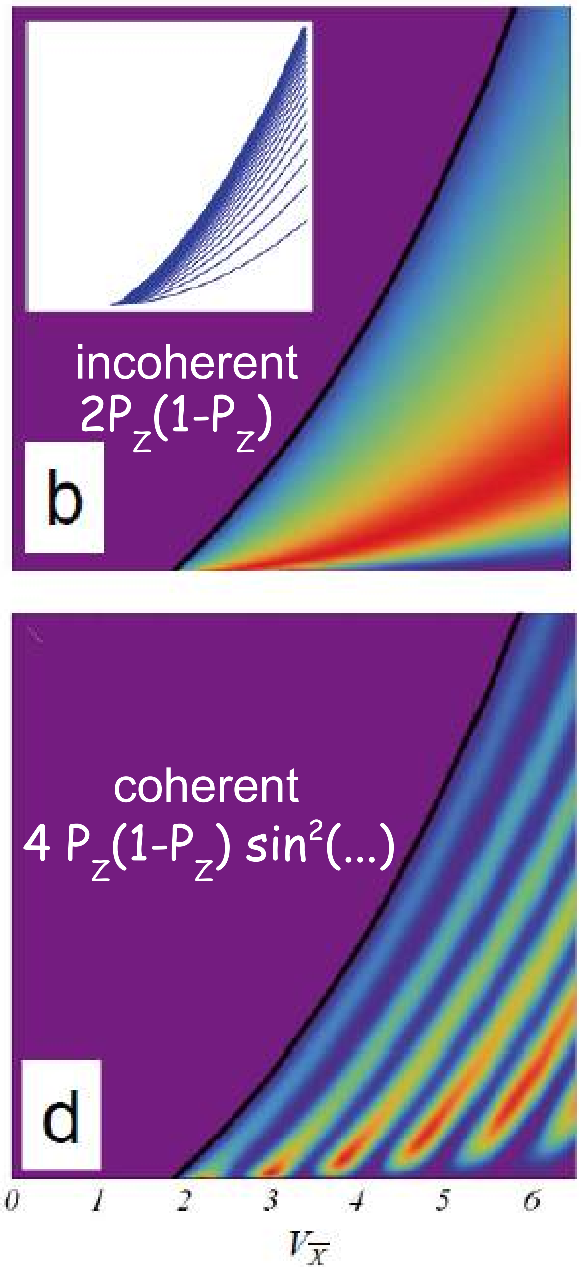
Two paths interferometer in energy space
Avoided crossings (gapped Dirac cones) act as beam splitters
Dynamical phase is different for the two paths

Stückelberg 1932





Coherent double Dirac cone



Combining probability amplitudes gives:

$$P_t = 4P_Z(1-P_Z)\sin^2(\frac{\varphi_{\text{dyn.}}}{2} + \varphi_{\text{St.}})$$

With the dynamical phase difference

$$\varphi_{\text{dyn.}} = \frac{1}{\hbar}\int_{-t_0}^{t_0} dt(E_+(t) - E_-(t))$$

and the Stokes phase (acquired upon reflection on the « beam splitter »)

$$\varphi_{\text{St.}} = \frac{\pi}{4} + \delta(\ln\delta - 1) + \arg\Gamma(1 - i\delta)$$

Stückelberg interferences give access to the energy spectrum via the dynamical phase.

Can we probe the eigenstates as well ?

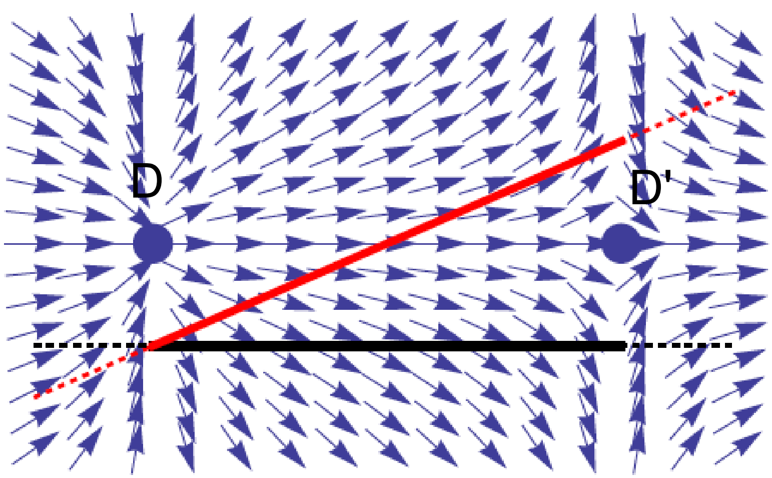
Lim, Fuchs & Montambaux PRL 2012

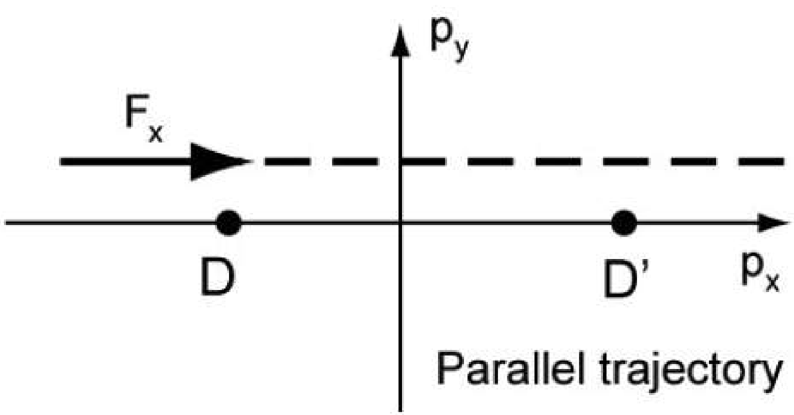
Phase of the Stückelberg interferometer

$$P_t = 4P_Z(1-P_Z)\sin^2(\frac{\varphi_{\mathrm{dyn.}} + \varphi_g}{2} + \varphi_{\mathrm{St.}})$$

1) Dynamical phase
2) Stokes phase
3) Extra contribution : geometrical phase (probes eigenstates), open-path Berry phase involving two bands

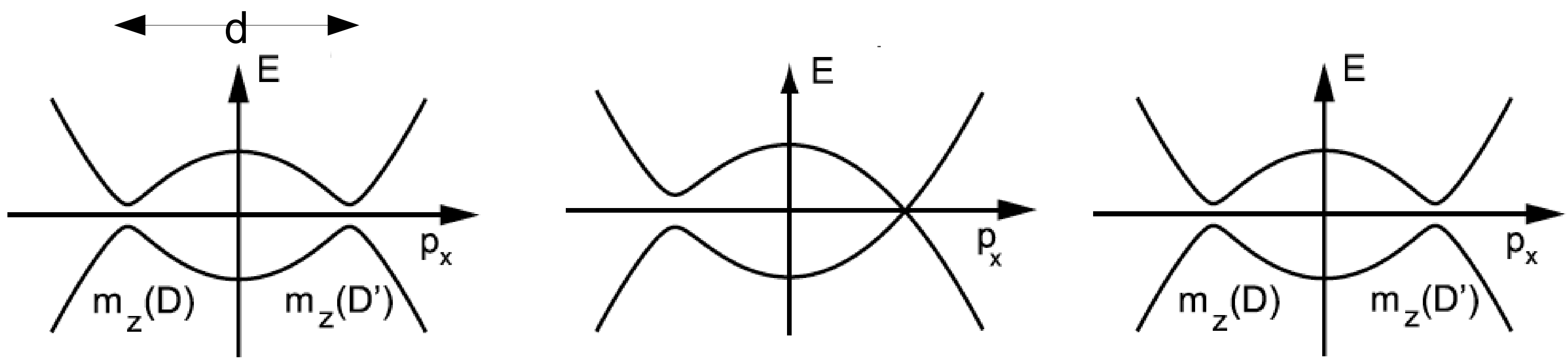
$$\varphi_g = \int_{-t_0}^{t_0} dt(\langle\psi_-|i\partial_t|\psi_-\rangle - \langle\psi_+|i\partial_t|\psi_+\rangle)$$
$$+ \arg\langle\psi_-(-t_0)|\psi_-(t_0)\rangle - \arg\langle\psi_+(-t_0)|\psi_+(t_0)\rangle$$





Lim, Fuchs & Montambaux PRL 2014

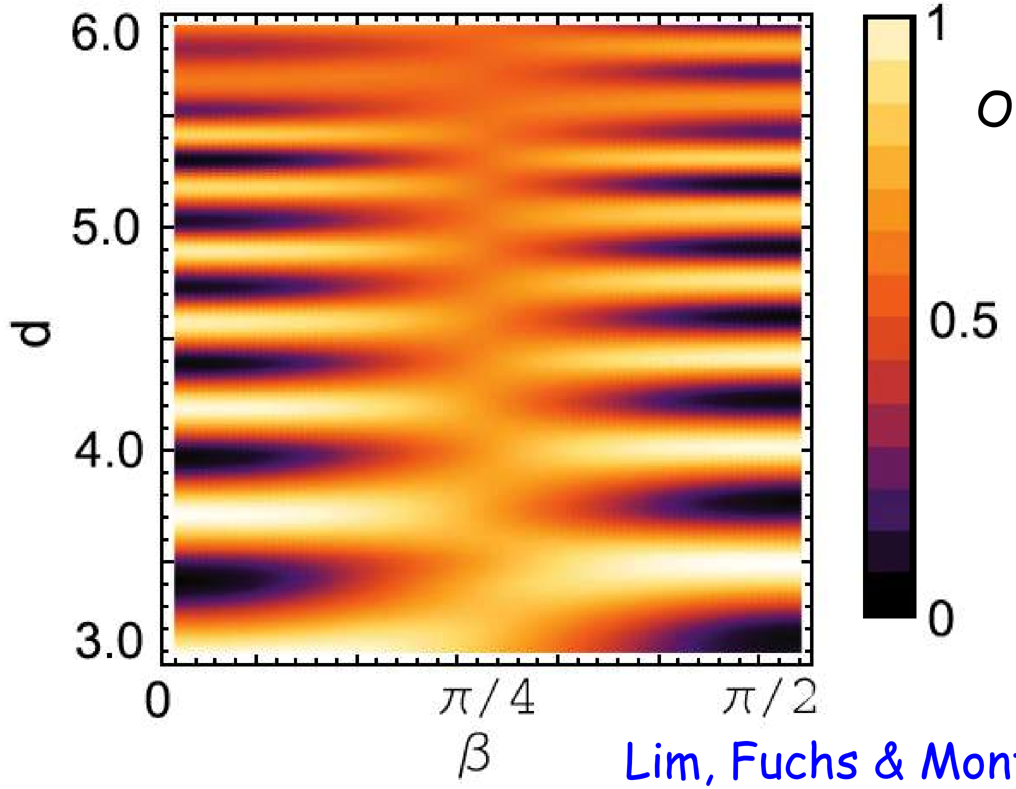
Stückelberg interferences for topol. insulators



Trivial Insulator Same mass (β=0)

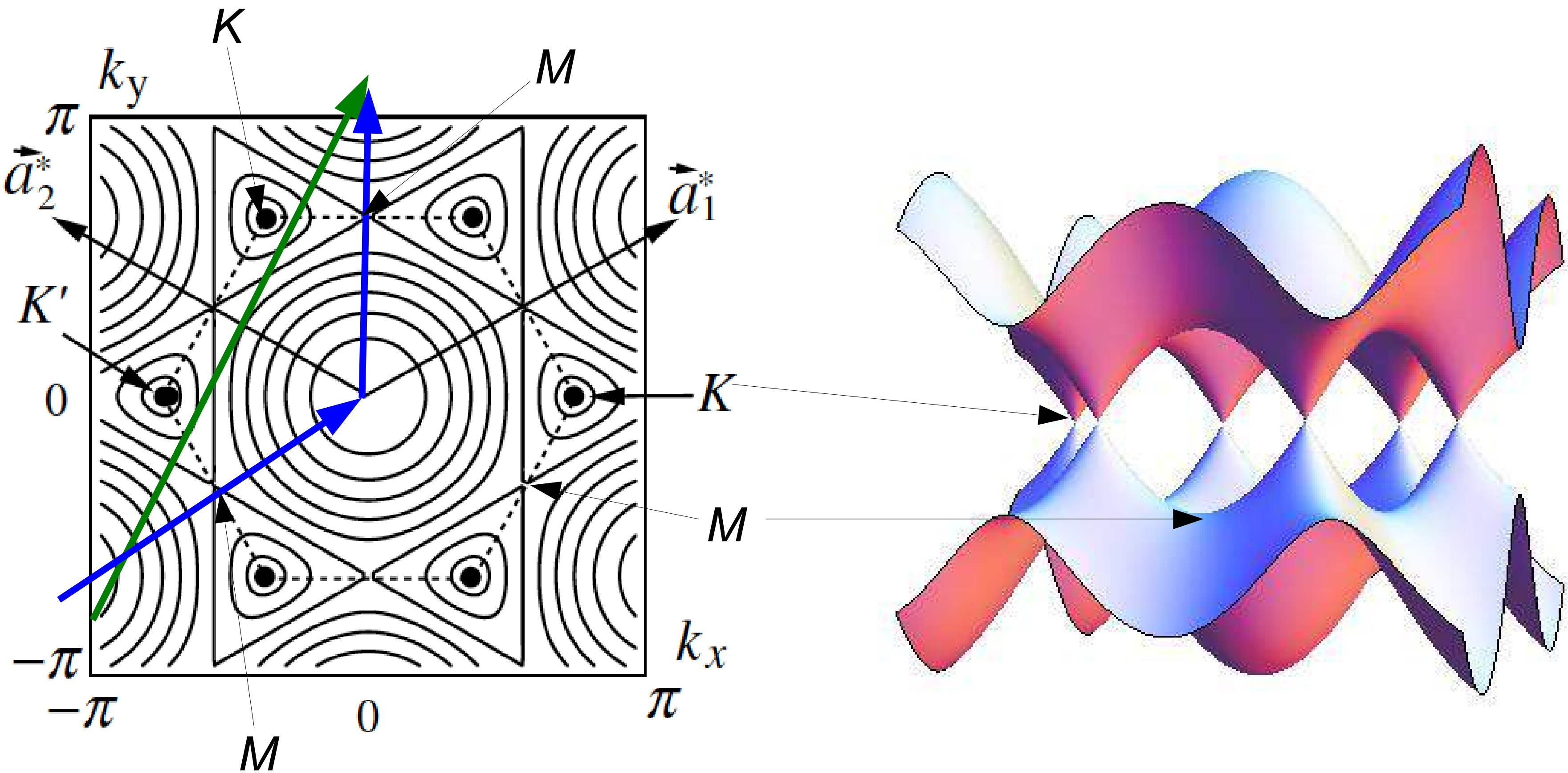
Singular semi-metal ($\beta=\pi/4$)

Topological Insulator Opposite mass ($\beta=\pi/2$)



Lim, Fuchs & Montambaux PRL 2014

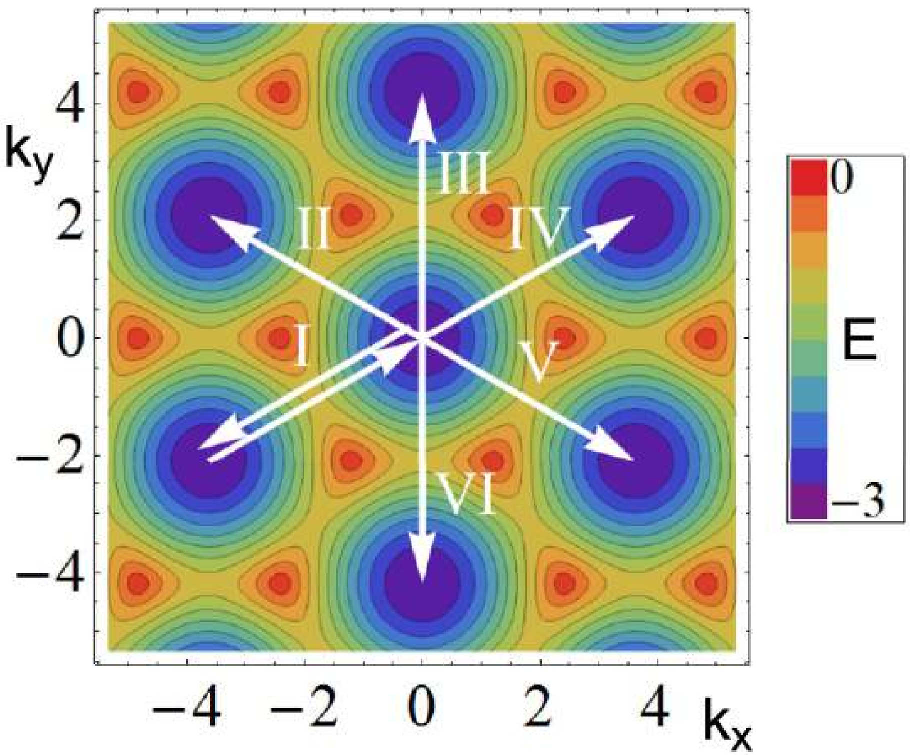
Double K point versus double M point
Interferometer with finite bandwidth

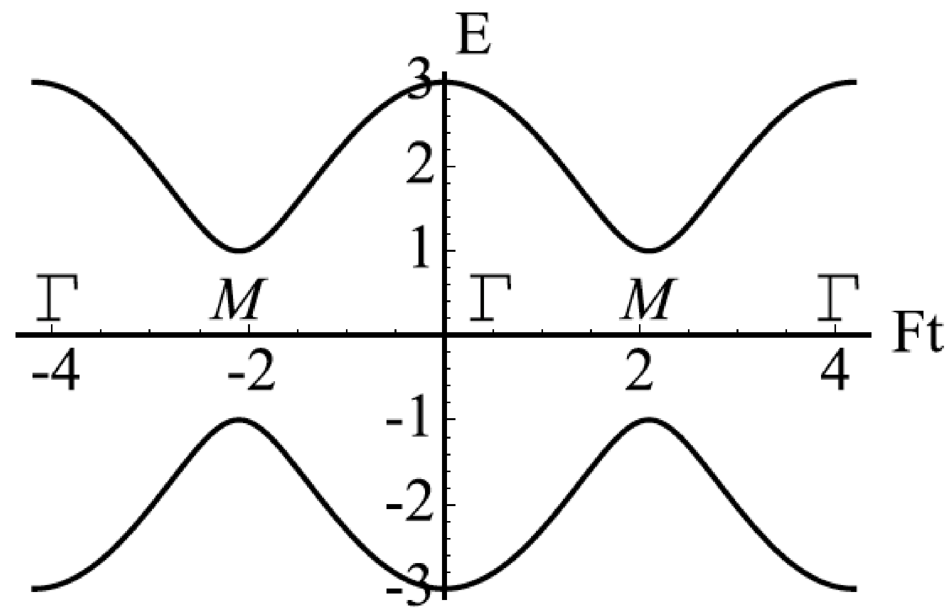


K points : Dirac points
Linearly vanishing DoS
Gapless
Vicinity of K points : small « lateral » gap
Zürich exp. with fermions

M points : saddle points
Van Hove singularity in DoS
Large « saddle point » gap
Münich exp. with bosons

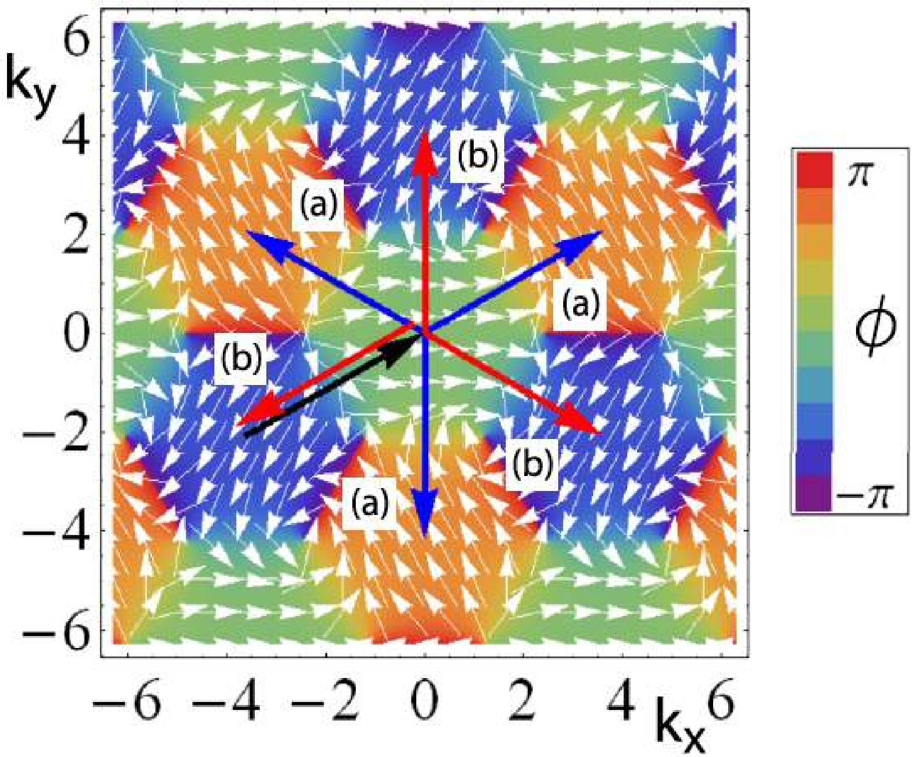
Ambulance cross :
Six trajectories & one energy landscape

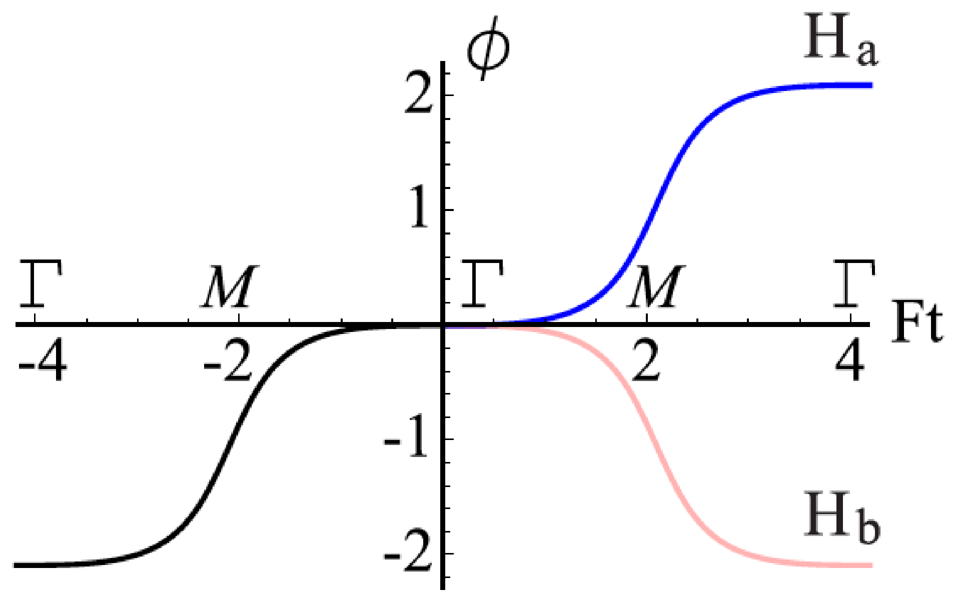




Units s.t. hopping amplitude J = 1, NN distance a = 1, ħ

Ambulance cross :
Six trajectories & two phase landscapes

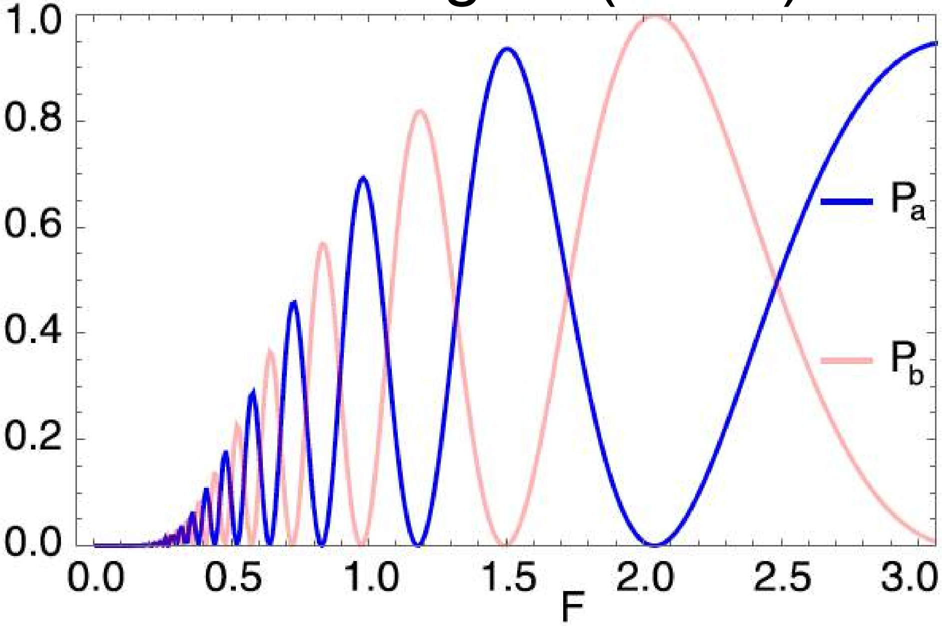




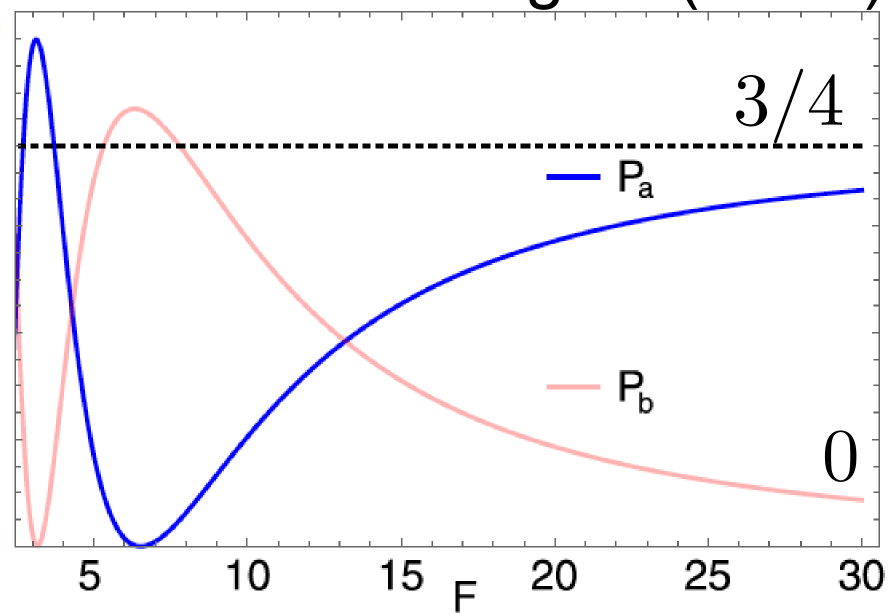
Units s.t. hopping amplitude J = 1, NN distance a = 1, ħ

Ambulance cross :
Two ≠ Stückelberg interferometers

Adiabatic region (F<<1)



Sudden region (F>>1)

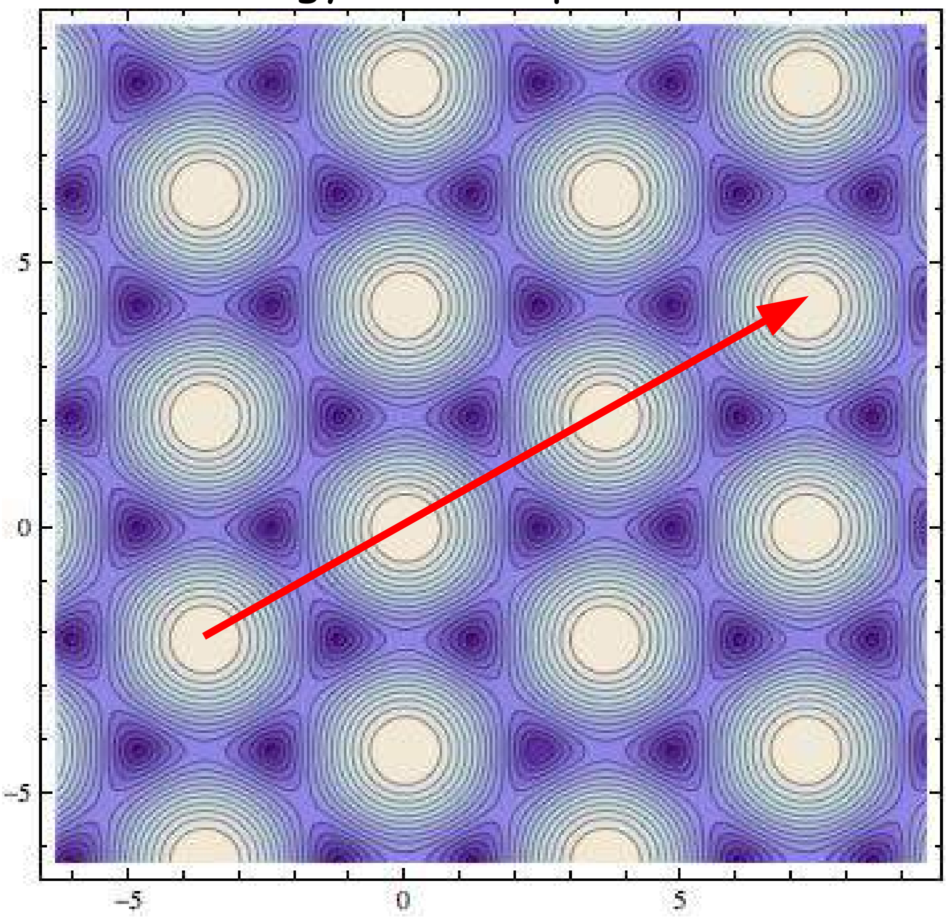


- Geometric π phase shift at any force F [in units of J/a]
phase opposition
- Saturation of proba to 0 or 3/4 in sudden limit
→ reveals extended periodicity beyond 1st BZ

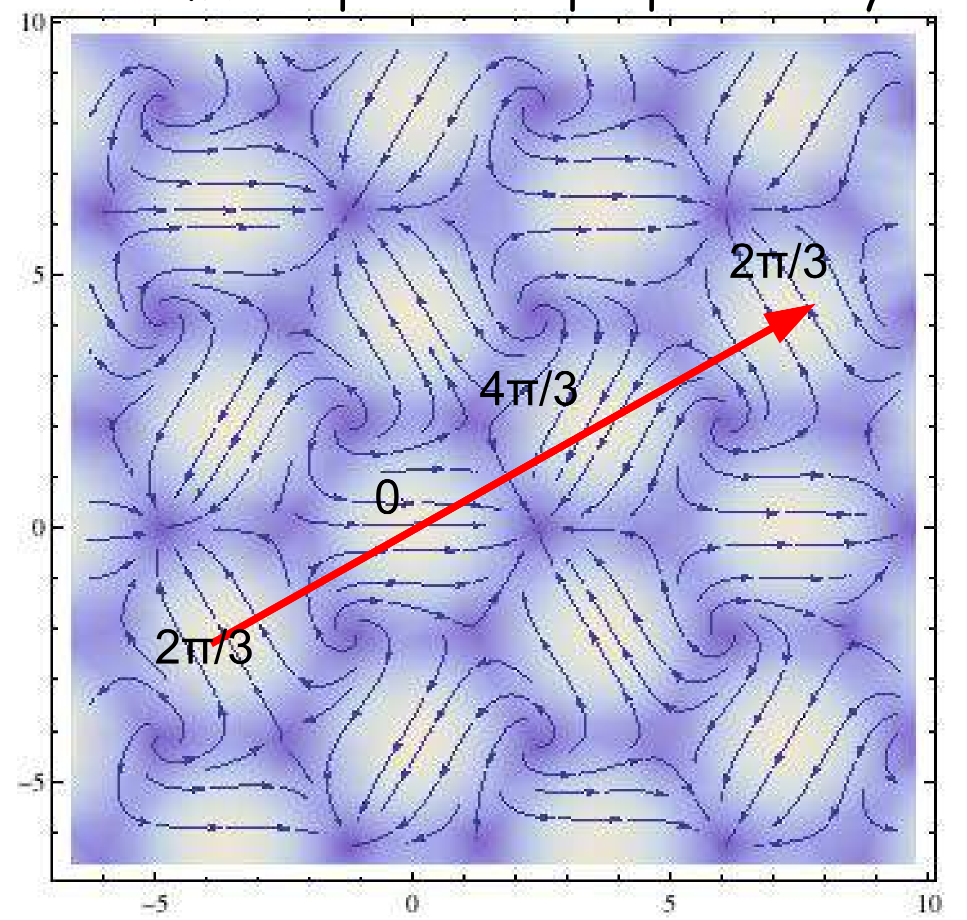
Fast and straight trajectory from Γ to Γ
Tripled periodicity H(k) ?

Interband transition proba sequence :
3/4, 3/4, 0, 3/4, 3/4, 0, etc ?

Iso-energy curves : periodic with BZ



Azimuthal phase : triple periodicity



Sudden approximation : overlap matrix

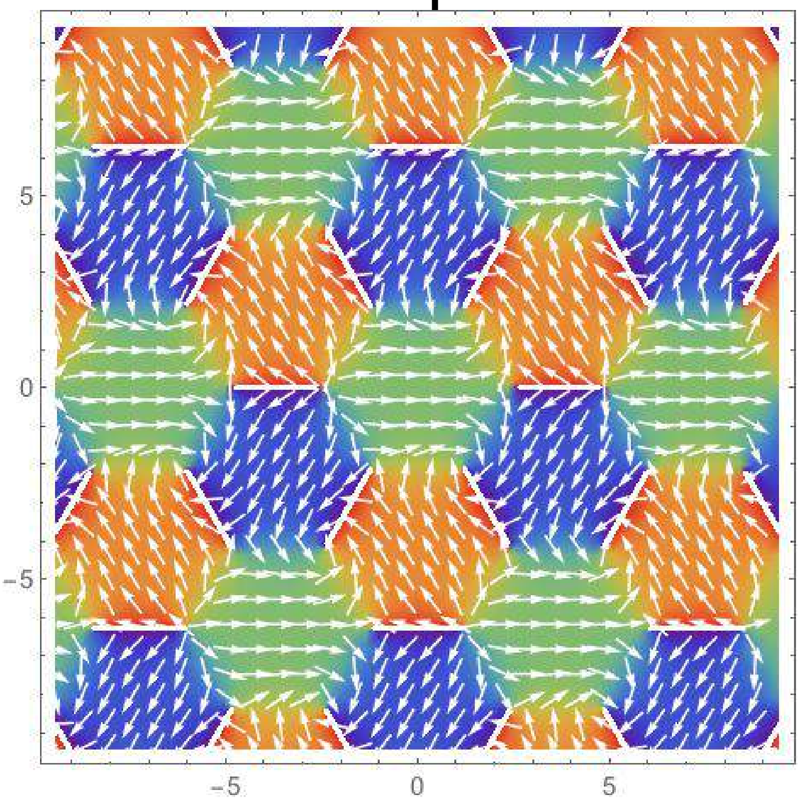
Interband transition probability

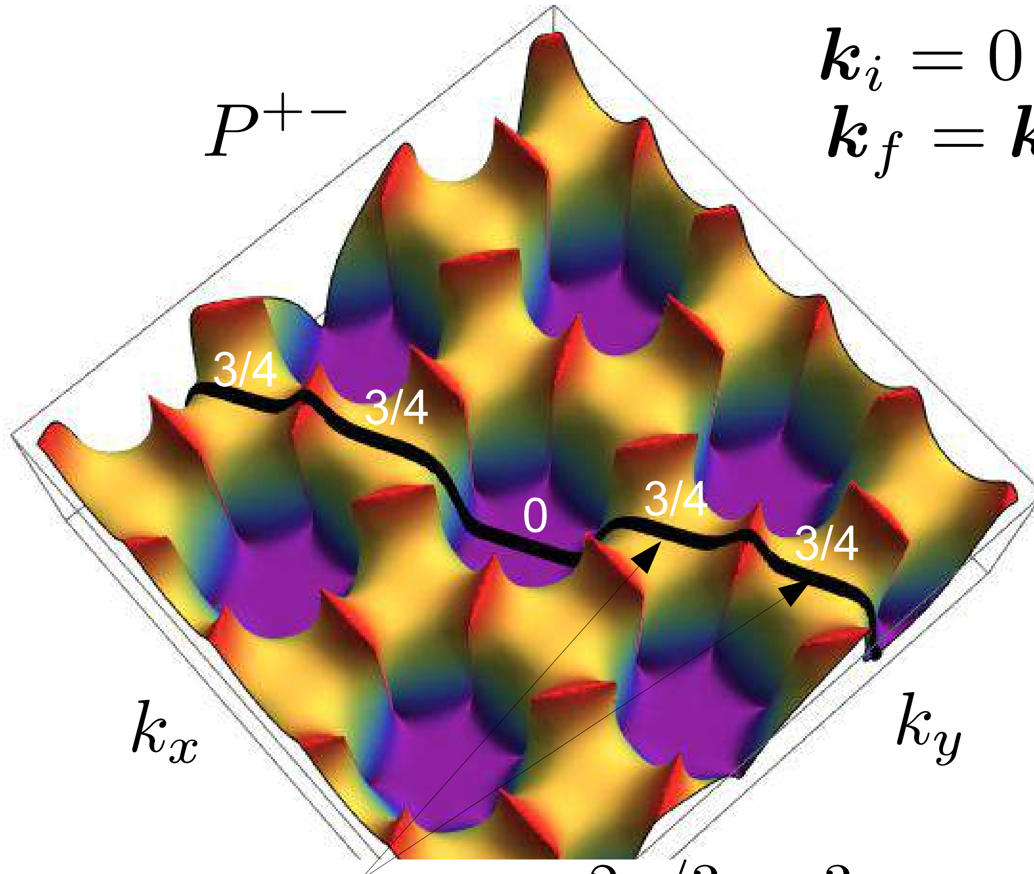
$$P^{+-}(t_f) = |\langle u_+(\boldsymbol{F}t_f)|u_-(\boldsymbol{F}t_i)\rangle|^2 = \sin^2\frac{\Delta\phi}{2}$$

$$\Delta\phi = \phi_f - \phi_i$$

$$\boldsymbol{k}_i = 0$$
$$\boldsymbol{k}_f = \boldsymbol{k}$$

Phase pattern



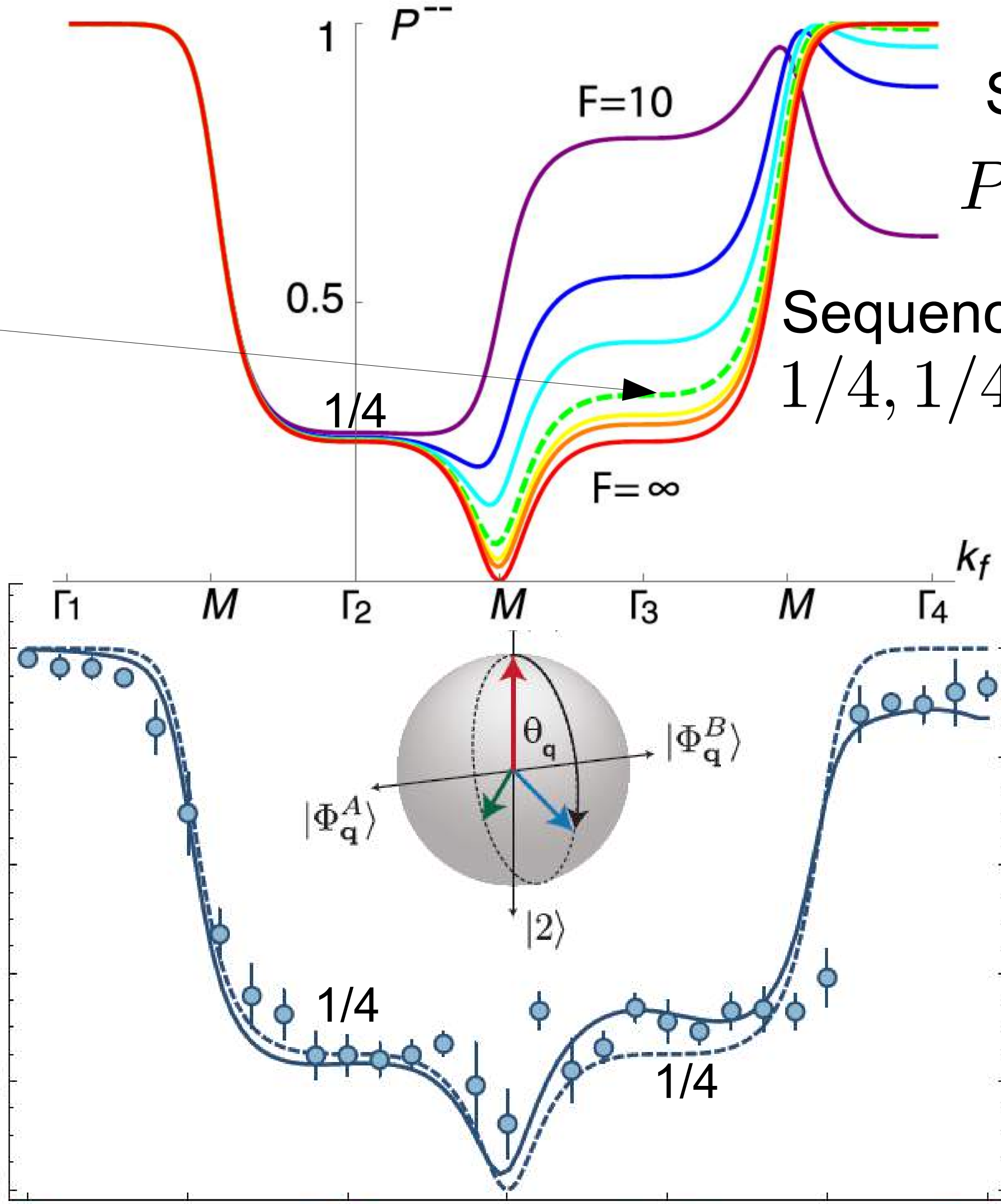


$$P^{+-} = \sin^2\frac{2\pi/3}{2} = \frac{3}{4}$$

Fast and straight trajectory from Γ to Γ

Theory : (F=30 dashed green)

Exp : (F=30)



Staying probability

$$P^{--} = 1 - P^{+-}$$

Sequence

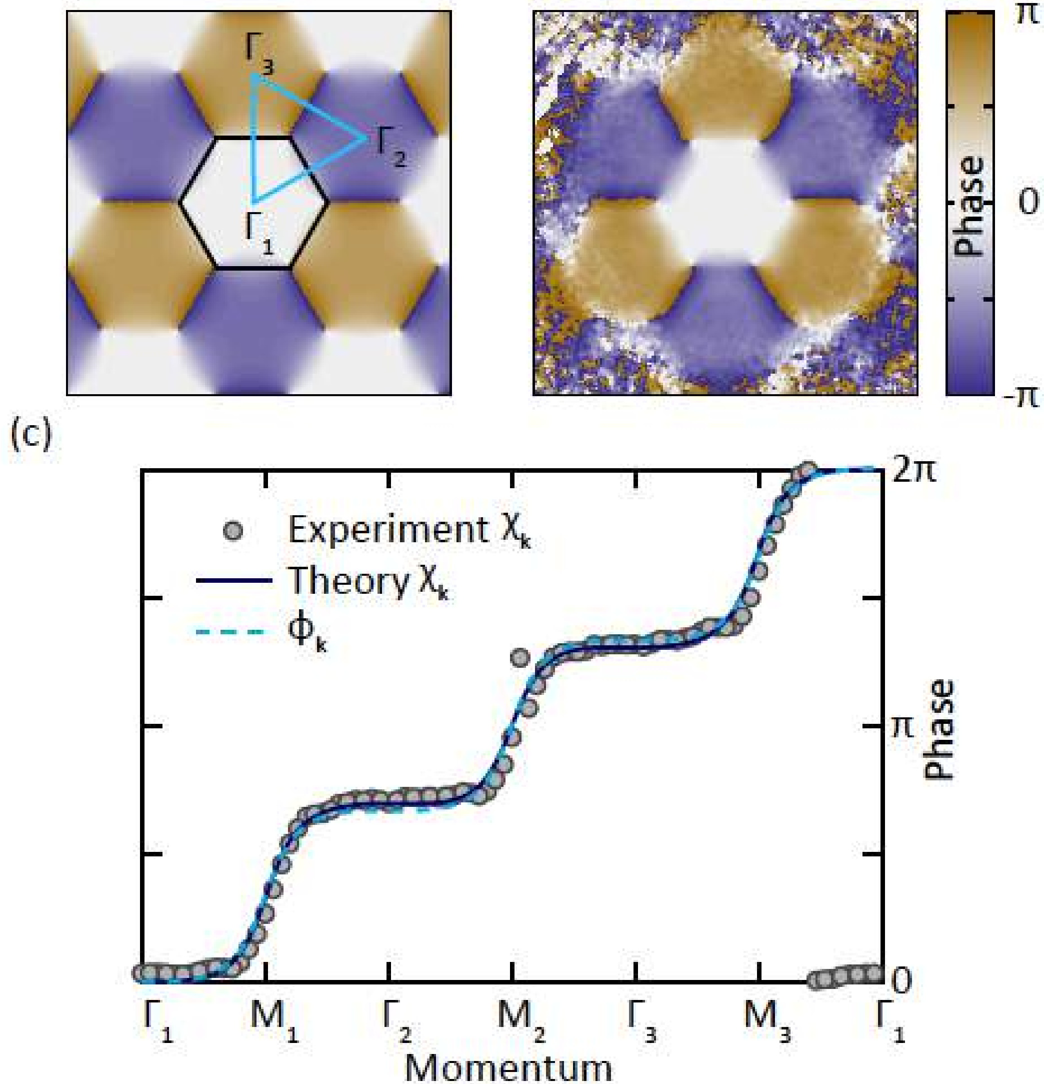
$$1/4, 1/4, 1, 1/4, 1/4, 1, \ldots$$

Force F in units of J/a

Exp : Li,…,Schneider, Science 2016

Theo : Lim, Fuchs, Montambaux, PRA 2015

Tripled periodicity of relative phase measured in honeycomb lattice

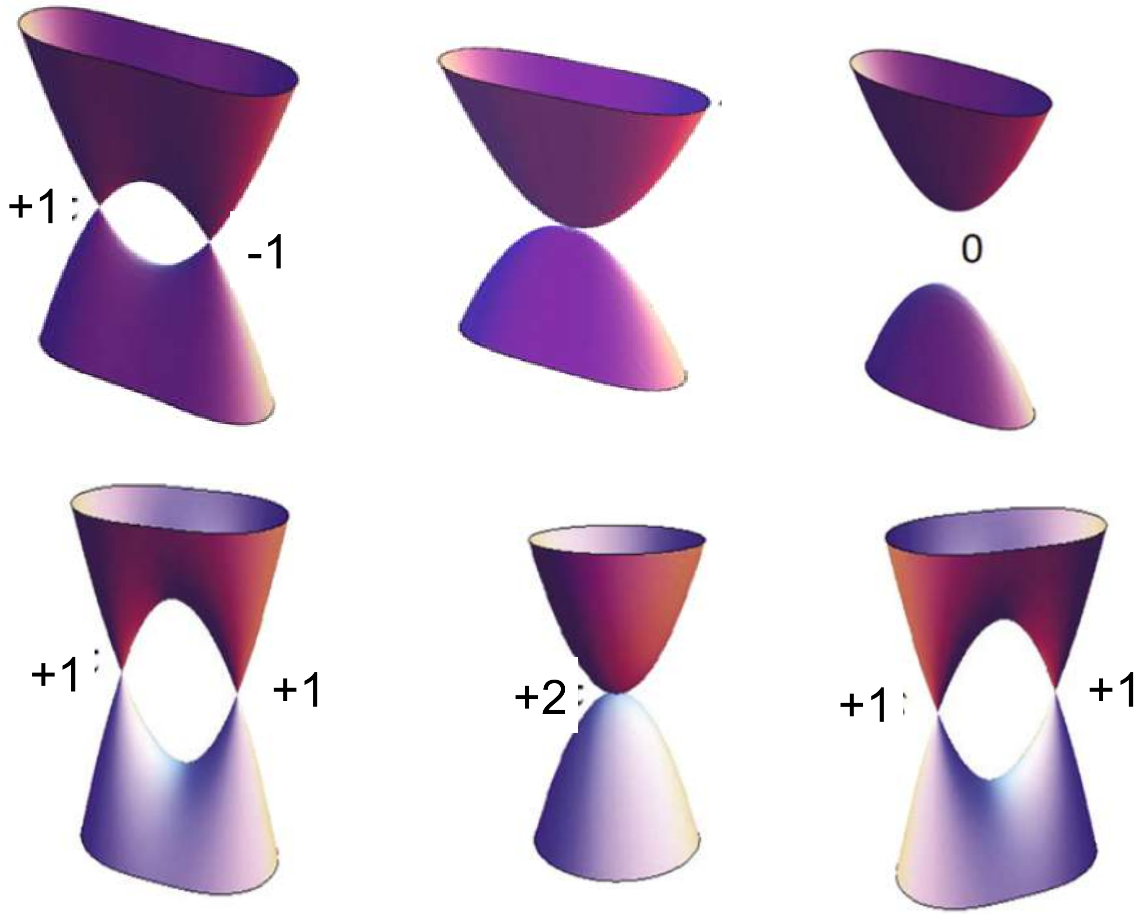


Tarnowski,…, Weitenberg, PRL 2017

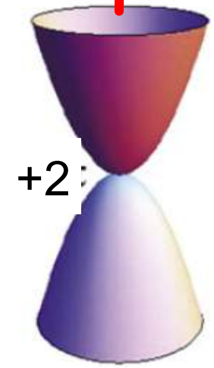
Conclusion :
Two Dirac points

+- scenario
(semi-Dirac)

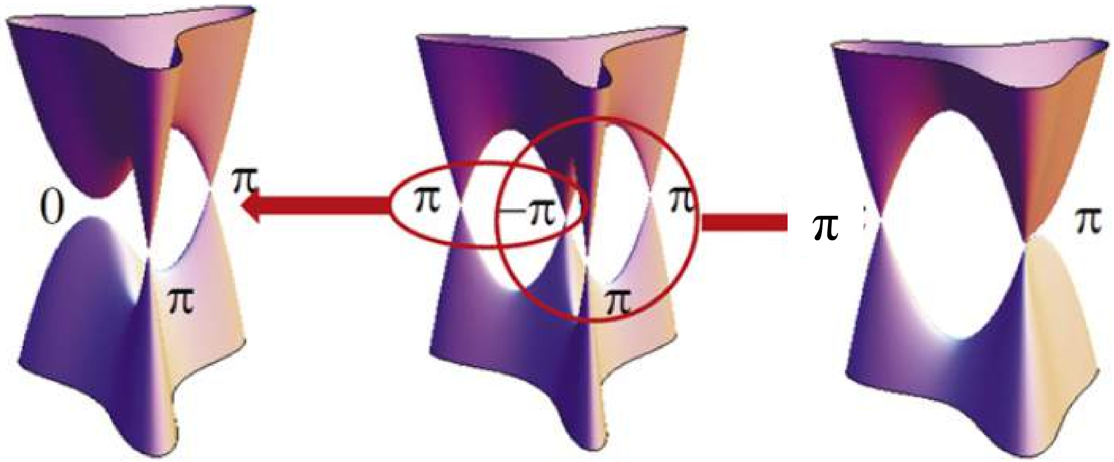
++ scenario
(nematic
instability
C_4 or $C_6 \rightarrow C_2$
QBCP W=+2)



Four Dirac points



+++- scenario
($C_6 \rightarrow C_3$ instability
QBCP W=+2)



++-- scenario
(QBCP W=0)